Can supersonic travel give rise to time paradoxes?

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Abstract

This article deals with three speculations: (i) Given space-time frames generated by clocks which use sound waves for synchronization; under what conditions would supersonic travel cause time paradoxes? (ii) How would creatures whose structural elements were based on sound waves avoid such time paradoxes? (iii) Is there a strategy to consistently circumvent limiting velocities such as the speed of sound or the speed of light?

time.tex

1 Operational concepts and intrinsic observers

John Casti posed the following scenario ([9]; for a related discussion, see [33]),

> *[[Imagine some creatures—]]they may be carbon-based creatures just like you and me. The difference is that they live in a world in which the primary sensory inputs are not from the electromagnetic spectrum like light, but rather come from sound waves. Note that this is not simply a world of the blind; rather, it is a world in which there are no sensory organs for perceiving any part of the electromagnetic spectrum. In this case, then, … such creatures would see the speed of sound as a fundamental barrier to the velocity of any material object. Yet we as creatures that* do *possess sensory organs for perceiving the electromagnetic spectrum see the sound barrier as no fundamental barrier, at all. So by analogical extension, there may be creatures "out there" who regard the speed of light as no more of a barrier than we regard the speed of sound.*

Casti's scenario is one in which the observers made out of sound waves are embedded in a universe, which, to them, consists solely out of sound waves. They develop an intrinsic description [6, 35, 25, 26, 31, 32, 33, 34] of this universe, which may appear drastically different (cf. [34], chapter 10) from what some hypothetical "super-observer" perceives by using, for instance, light. One of the first researchers bothering about these issues has been the 18th century physicist Boskovich ([6], rediscovered by Rössler [26], p. 143),

> *… And we would have the same impressions if, under conservation of distances, all directions would be rotated by the same angle, … And even if the distances themselves would be decreased, whereby the angles and the proportions would be conserved, …: even then we [[the observers]] would have no changes in our impressions. … A movement, which is common to us [[the observers]] and to the Universe, cannot be observed by us; not even if everything would be stretched or shrinked by an arbitrary amount.*

Although generally not presented that way, relativity theory consists of two distinct parts: (i) it deals with conventions about how to operationalize certain concepts such as "equal time at spatially separated points," in particularly synchronization procedures; and (ii) it states physical assumptions about the invariance of certain phenomena, such as the speed of light, and the laws of theoretical physics under changes of reference frames. Whereas the former conventions [ad (i)] are mere working definitions, the latter statements [ad (ii)] are supposed to be God-given, eternal symmetries. Conventions can be changed at the price of complicating the theoretical formalism by means of a non-optimal representation of theory. Phenomena are a matter of physical fact. For instance, a preferred frame of reference, e.g., the one at rest with respect to cosmic background radiation, could be artificially introduced. To put it in the words of the late John Bell (cf. [3], p. 34; for a related discussion, see [18]):

> *You can pretend that whatever inertial frame you have chosen is the ether of the 19th century physicists, and in that frame you can confidently apply the idea of the FitzGerald contraction [[of length of material bodies such as scales]], Larmor [[time]] dilation and Lorentz lag. It is a great pity that students dont understand this. …*

Preferred frames of reference can also be introduced by abandoning Einstein's conventions for clock synchronization and by using "absolute" Galilei spacetime co-ordinates. Clocks in moving frames (with respect to the preferred frame) would show the "absolute" time which characterizes the "absolute" Galileian frame. As a result, the value of the speed of light would become frame dependent. The electromagnetic laws would have to be represented in a non-covariant manner. But despite the overhead due to non-optimal representation, the physics would not change.

It was the radical operational approach in Einstein's theory of special relativity, which stimulated the physicist Bridgman. In particular, Bridgman demanded that the meaning of theoretical concepts should ultimately be based on concrete physical operations. That is, (cf. [8], p. V),

> *the meaning of one's terms are to be found by an analysis of the operations which one performs in applying the term in concrete situations or in veryfying the truth of statements or in finding the answers to questions.*

More specifically (cf. [7], p. 103),

> *… the meaning of length is to be sought in those operations by which the length of physical objects is determined, and the meaning of simultaneity is sought in those physical operations by which it is determined whether two physical events are simultaneous or not.*

Therefore, we are free to choose whatever physical concepts seem appropriate as long as they are operational. In particular, Casti's sound creatures might choose sound, the author's water-fleas may use water waves, and human physicists may use light for the purpose of Einstein synchronization. The resulting frame of references will be different in all these cases. The space-time parameters by which events and phenomena are represented will be different, too. Using different types of formal representations of the same physical system—one may speak of layers or levels of description—in order to grasp different features of that system associated, for instance, with different levels of complexity, resembles Anderson's thesis of "emerging laws" ([1], p. 193; cf. Schweber [28]),

> *The ability to reduce everything to simple fundamental laws does not imply the ability from these laws and reconstruct the universe. … The constructionist hypothesis breaks down when confronted with the twin difficulties of scale and complexity. … at each level of complexity, entirely new properties appear, and the understanding of the new behaviors requires research which I think is as fundamental in nature as any other.*

It is to be expected that in such an organization of physical concepts, the notion of causality need not be consistently defined for all descriptions *combined:* what may appear as causal connection (cause and effect) in one description needs not be causally connected at all in another description. Therefore it is of great relevance to make precise the limits and applicability of each of these descriptions.

Next, an example for the explicit construction of such reference frames is given in the Cellular Automaton (CA) context is given.

2 Space-time frames in Cellular Automata

In what follows, an explicit example for the construction of space-time frames in computer-generated universes which are generated by *intrinsic* procedures and observations will be given. The onedimensional Cellular Automaton (CA) models [39, 16, 17, 36, 40] considered here are equivalent to any other universal computing agent but have the advantage of good representability on the twodimensional printing page combined with easy programmability.

Our primary concern will be the explicit construction of intrinsic space-time frames by adopting Einstein's synchronization conventions. Pointedly stated, we are interested in virtual reality and physical epistemology, not physics. We do *not* attempt to reconstruct relativity theory one-to-one in the cellular automaton context. In particular, no Lorentz-invariant kinematic theory is introduced. Therefore, certain physical statements, in particular the relativity principle, stating that all laws of physics have an identical form in all inertial frames, needs not to be satisfied.

However, it has been argued for quite some time [16] that, since all "construable" (in the sense of "recursively enumerable") universes are realisable within the CA framework, also the special theory of relativity can be implemented on such a structure [37]. Kinematic questions have been taken into account, for instance, by Bialynicki-Birula [4] and Smith [29]. Both approaches — the epistemic presented here and the kinematic — fit together well and may lead to a complete modeling of Lorentz invariance in cellular automaton structures.

Informally speaking, CA are a "large (usually an infinite) collection" of interconnected finite automata. Each one of these finite automata is being thought of as a cell, which is locally connected to its immediate neighbours. I.e., the evolution of each one of the cells forming the CA depends on its own state and the states of its immediate neighbour cells.

More precisely, assume discrete time cycles, labelled by $0, 1, 2, \ldots$. A CA is an infinite collection of locally connected finite automata (cells) of the tessellated onedimensional line. Let $a_{i,n}(t)$ stand for the i'th internal state of the n'th automaton (cell), $n = (n_1, n_2, \ldots, n_D)$, at time t. Let $\{n\}$ characterise the neighbourhood of n, including n, and let $a_{i,\{n\}}(t)$ stand for the internal states of the automaton cells around n, including the origin n. The internal state $a_{i,n}(t+1)$ of the automaton at n at time $t+1$ is given by a computable function φ of the state and its neighbourhood states at time t, i.e.,

$$a_{i,n}(t+1) = \varphi(a_{i,\{n\}}(t)) \quad . \tag{1}$$

2.1 Synchronization

Without loss of generality it is assumed that the maximal velocity, denoted by c, a body of information can move is one cell per cycle time. Flows of this kind will be called *rays*. In analogy to relativity theory, this velocity can be used to define clocks or synchronized events. We shall start by an explicit model of a ray clock. It consists of two mirrors, denoted by `I`, and a ray of velocity 1 cell per time cycle, denoted by > and <, which is constantly reflected back and forth between the two mirrors, and a one-place digital display right to the right mirror. In this model, after each backreflection of the light ray, the digit on the display increases by one modulus 10. The explicit transformation rules for a onedimensional CA with these properties are listed in appendix A. The time evolution of a ray clock is drawn in Fig. 1.

A very similar configuration as for the light clock can be used as a device for Einstein synchronization [12, 34]: Assume two clocks at two arbitrary points A and B which are "of similar kind." At some arbitrary A-time t_A a ray goes from A to B. At B it is instantly (without delay) reflected at B-time t_B and reaches A again at A-time $t_{A'}$. The clocks in A and B are *synchronized* if

$$t_B - t_A = t_{A'} - t_B \quad . \tag{2}$$

The two-ways ray velocity is given by

$$\frac{2|AB|}{t_{A'} - t_A} = c \quad , \tag{3}$$

where $|AB|$ is the distance between A and B.

The ray velocity can then be *defined* to be identical for all frames, irrespective of whether they are moving with respect to the rest frame of the cellular space or not. Of course, this invariance of the ray

```
... __I>__I0__ ...      ... __I>__I4__ ...      ... __I>__I8__ ...
... __I_>_I0__ ...      ... __I_>_I4__ ...      ... __I_>_I8__ ...
... __I__>I0__ ...      ... __I__>I4__ ...      ... __I__>I8__ ...
... __I__<*0__ ...      ... __I__<*4__ ...      ... __I__<*8__ ...
... __I_<_I1__ ...      ... __I_<_I5__ ...      ... __I_<_I9__ ...
... __I<__I1__ ...      ... __I<__I5__ ...      ... __I<__I9__ ...
... __I>__I1__ ...      ... __I>__I5__ ...      ... __I>__I9__ ...
... __I_>_I1__ ...      ... __I_>_I5__ ...      ... __I_>_I9__ ...
... __I__>I1__ ...      ... __I__>I5__ ...      ... __I__>I9__ ...
... __I__<*1__ ...      ... __I__<*5__ ...      ... __I__<*9__ ...
... __I_<_I2__ ...      ... __I_<_I6__ ...      ... __I_<_I0__ ...
... __I<__I2__ ...      ... __I<__I6__ ...      ... __I<__I0__ ...
... __I>__I2__ ...      ... __I>__I6__ ...      ... __I>__I0__ ...
... __I_>_I2__ ...      ... __I_>_I6__ ...      ... __I_>_I0__ ...
... __I__>I2__ ...      ... __I__>I6__ ...      ... __I__>I0__ ...
... __I__<*2__ ...      ... __I__<*6__ ...      ... __I__<*0__ ...
... __I_<_I3__ ...      ... __I_<_I7__ ...      ... __I_<_I1__ ...
... __I<__I3__ ...      ... __I<__I7__ ...      ... __I<__I1__ ...
... __I>__I3__ ...      ... __I>__I7__ ...              .
... __I_>_I3__ ...      ... __I_>_I7__ ...              .
... __I__>I3__ ...      ... __I__>I7__ ...              .
... __I__<*3__ ...      ... __I__<*7__ ...
... __I_<_I4__ ...      ... __I_<_I8__ ...
... __I<__I4__ ...      ... __I<__I8__ ...
```

Figure 1: Ray clock.

speed with respect to changes of coordinate systems should ultimately be motivated by phenomenology and a proper choice of conventions. E.g., in relativity theory, the invariance of the Maxwell equations with respect to conformal, or angle preserving, coordinate transformations in four dimensions assures that for light-like vectors, $ds^2 = dx^2 - (c\,dt)^2 = 0$ and thus $x = ct$.

For synchronization, the same CA transformation rules as for the ray clock, which are listed in appendix A can be used. In Fig. 2, an example of synchronization between two clocks A and B is drawn.

2.2 Moving reference frames

This section shall be concerned with what happens with the intrinsic synchronization and the space-time coordinates when observers are considered which move with respect to the CA medium. For simplicity, assume constant motion of v automaton cells per time cycle. With these units, the ray speed is $c = 1$, and $v \leq 1$.

There are numerous ways to simulate sub-ray motion on a CA. In what follows, the case $v = 1/3$ will be studied in such a way that every three CA time cycles the walls, symbolised by `I`, move one cell to the right. [Strictly speaking, there should be a periodic transformation of the wall such that $\mathtt{I} \rightarrow a \rightarrow b \rightarrow \mathtt{I}$ and the states a and b have the same reflection properties as `I`.]

Notice that two clocks which are synchronized in a reference frame which is at rest with respect to the CA medium are *not synchronized* in their own co-moving reference frame. Consider, as an example, the CA drawn in Fig. 3(a). (Strictly speaking, the CA rule here depends on a two-neighbor interaction.) By evaluating equation (2) for $t_A = 1$, $t_B = 4$, $t_{A'} = 5$, and $4 - 1 \neq 5 - 4$. If the first clock is corrected to make up for the different time of ray flights as in Fig. f:3(b), $t_A = 2$, $t_B = 4$, $t_{A'} = 6$, and $4 - 2 = 6 - 4$. This correction, however, is the reason for asynchronicity of the two ray clocks with respect to the "original" CA medium.

Let us now explicitly construct the coordinate axes $\bar{x}$ and $\bar{t}$ of a system which moves with constant velocity with respect to the medium. For convenience, let us switch to continuous coordinates. That is, the "grainyness" of the CA medium is disregarded; i.e., coordinates will be represented by real numbers. In this construction, the convention of the constancy of the speed of rays for all reference frames will play an important role. The new space and time axes will both become rotated towards the ray coordinates.

Consider Fig. 4, which represents the process drawn in Fig. 3(b). Assume that the two mirrors are a (arbitrary) units apart. For the system at rest with respect to the CA medium, a ray is emitted from the origin $A = (0, 0)$ and arrives at the mirror at $B = (vt_B + a, t_B)$, where it is reflected and arrives at the original mirror at $A' = (vt_{A'}, t_{A'})$. Since B lies on the ray which comes from the origin, $B = (t_B, t_B) = \left(\frac{a}{1-v}, \frac{a}{1-v}\right)$.

```
      clock1  A       B  clock2

... __I>__I0__I>______I__I>__I0__ ...
... __I_>_I0__I_>_____I__I_>_I0__ ...
... __I__>I0__I__>____I__I__>I0__ ...
... __I__<*0__I___>___I__I__<*0__ ...
... __I_<_I1__I____>__I__I_<_I1__ ...
... __I<__I1__I_____>_I__I<__I1__ ...
... __I>__I1__I______>I__I>__I1__ ...
... __I_>_I1__I______<*__I_>_I1__ ...
... __I__>I1__I_____<_I__I__>I1__ ...
... __I__<*1__I____<__I__I__<*1__ ...
... __I_<_I2__I___<___I__I_<_I2__ ...
... __I<__I2__I__<____I__I<__I2__ ...
... __I>__I2__I_<_____I__I>__I2__ ...
... __I_>_I2__I<______I__I_>_I2__ ...
... __I__>I2__I>______I__I__>I2__ ...
... __I__<*2__I_>_____I__I__<*2__ ...
... __I_<_I3__I__>____I__I_<_I3__ ...
... __I<__I3__I___>___I__I<__I3__ ...
... __I>__I3__I____>__I__I>__I3__ ...
... __I_>_I3__I_____>_I__I_>_I3__ ...
... __I__>I3__I______>I__I__>I3__ ...
    .
    .
    .
```

Figure 2: Synchronization by ray exchange.

Since A' lies on the ray which comes from B, $t_{A'} = -x_{A'} + C$ and $t_B = -x_B + C$. By evaluating C, one obtains $A' = \left(\frac{2va}{1-v^2}, \frac{2a}{1-v^2}\right)$. In the coordinate system which is moving with respect to the CA medium, $\overline{A} = (0, 0)$. In order for the clocks to be synchronized, i.e., by equation (2), $\bar{t}_B = (\bar{t}_{A'} + \bar{t}_A)/2$. But this should also be the time coordinate $\bar{t}_1$, since this is just half the time from A to A'. Thus one obtains two points (events) $1 = \left(\frac{vt_{A'}}{2}, \frac{t_{A'}}{2}\right)$ and B whose time coordinates in the moving reference frame is identical; i.e., $\bar{t}_1 = \bar{t}_B$. A short calculation shows that, with respect to the coordinate system which is at rest in the CA medium, the lines of equal time coordinates for a system wich is moving with constant velocity v, e.g., the $\bar{x}$-axis, have slope $1/v$, whereas the lines of equal space coordinates, e.g., the $\bar{t}$-axis, have slope v.

These transformation of coordinate axes correspond to the Lorentz transformations

$$\bar{t} = \frac{t - vx}{\sqrt{1-v^2}} \quad , \quad \bar{x} = \frac{x - vt}{\sqrt{1-v^2}} \quad . \tag{4}$$

The specific form of the transformation (4) comes as no surprise, since it has been derived by implicitly assuming that constant motion transforms into constant motion and that the ray speed is the same for all reference frames; both conditions being the kinematic equivalent to the relativity principle.

2.3 Sub-ray synchronization

So far, only rays propagating one CA cell per cycle have been considered. It is not entirely unreasonable to assume synchronization with signals which propagate slower than these rays.

A typical example would be a signal with one-half ray speed, i.e., $c' = 1/2$. Assume that, in addition to the space-time frames generated by ray exchange ($c = 1$), a different set of frames is generated by signalling with one-half ray speed ($c' = 1/2$). If one compares space-time scales which are generated by such signals with the space-time scales generated by ray exchange at $c = 1$, one obtains equivalence only for one frame of reference; e.g., the one at rest with respect to the CA medium. For all other reference frames, synchronization and thus coordinate transformations would be different.

One consequence of sub-ray synchronization is the possibility of “super-ray” speed $v > 1/2$, and of a “time travel” with respect to such space-time frames. I.e., for certain observers moving with $v > 1/2$, the time coordinate defined by $c' = 1/2$ would “run backward.” This “time travel” however, is nothing mysterious, but the outcome of the specific synchronization convention chosen. See the next section for a further discussion.

```
__I>_I0__I___<___I__I>_I0________________
__I_>I0__I__<____I__I_>I0________________
__I_<*0__I_<_____I__I_<*0________________
___I>_I1__I>______I__I>_I1_______________
___I_>I1__I_>_____I__I_>I1_______________
___I_<*1__I__>____I__I_<*1_______________
____I>_I2__I__>____I__I>_I2______________
____I_>I2__I___>___I__I_>I2______________
____I_<*2__I____>__I__I_<*2______________
_____I>_I3__I____>__I__I>_I3_____________
_____I_>I3__I_____>_I__I_>I3_____________
_____I_<*3__I______>I__I_<*3_____________
______I>_I4__I______>I__I>_I4____________
______I_>I4__I______<*__I_>I4____________
______I_<*4__I_____<_I__I_<*4____________
_______I>_I5__I___<___I__I>_I5___________
_______I_>I5__I__<____I__I_>I5___________
_______I_<*5__I_<_____I__I_<*5___________
________I>_I6__I>______I__I>_I6__________
________I_>I6__I_>_____I__I_>I6__________
________I_<*6__I__>____I__I_<*6__________
_________I>_I7__I__>____I__I>_I7_________
_________I_>I7__I___>___I__I_>I7_________
_________I_<*7__I____>__I__I_<*7_________
__________I>_I8__I____>__I__I>_I8________
__________I_>I8__I_____>_I__I_>I8________
__________I_<*8__I______>I__I_<*8________
___________I>_I9__I______>I__I>_I9_______
___________I_>I9__I______<*__I_>I9_______
___________I_<*9__I_____<_I__I_<*9_______
____________I>_I0__I___<___I__I>_I0______
____________I_>I0__I__<____I__I_>I0______
____________I_<*0__I_<_____I__I_<*0______
_____________I>_I1__I>______I__I>_I1_____
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_____________I_<*1__I__>____I__I_<*1_____
______________I>_I2__I__>____I__I>_I2____
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______________I_<*2__I____>__I__I_<*2____
_______________I>_I3__I____>__I__I>_I3___
_______________I_>I3__I_____>_I__I_>I3___
_______________I_<*3__I______>I__I_<*3___
________________I>_I4__I______>I__I>_I4__
________________I_>I4__I______<*__I_>I4__
________________I_<*4__I_____<_I__I_<*4__
            .
            .
            .

(a)
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__I>_I1__I___<___I__I>_I0________________
__I_>I1__I__<____I__I_>I0________________
__I_<*1__I_<_____I__I_<*0________________
___I>_I2__I>______I__I>_I1_______________
___I_>I2__I_>_____I__I_>I1_______________
___I_<*2__I__>____I__I_<*1_______________
____I>_I3__I__>____I__I>_I2______________
____I_>I3__I___>___I__I_>I2______________
____I_<*3__I____>__I__I_<*2______________
_____I>_I4__I____>__I__I>_I3_____________
_____I_>I4__I_____>_I__I_>I3_____________
_____I_<*4__I______>I__I_<*3_____________
______I>_I5__I______>I__I>_I4____________
______I_>I5__I______<*__I_>I4____________
______I_<*5__I_____<_I__I_<*4____________
_______I>_I6__I___<___I__I>_I5___________
_______I_>I6__I__<____I__I_>I5___________
_______I_<*6__I_<_____I__I_<*5___________
________I>_I7__I>______I__I>_I6__________
________I_>I7__I_>_____I__I_>I6__________
________I_<*7__I__>____I__I_<*6__________
_________I>_I8__I__>____I__I>_I7_________
_________I_>I8__I___>___I__I_>I7_________
_________I_<*8__I____>__I__I_<*7_________
__________I>_I9__I____>__I__I>_I8________
__________I_>I9__I_____>_I__I_>I8________
__________I_<*9__I______>I__I_<*8________
___________I>_I0__I______>I__I>_I9_______
___________I_>I0__I______<*__I_>I9_______
___________I_<*0__I_____<_I__I_<*9_______
____________I>_I1__I___<___I__I>_I0______
____________I_>I1__I__<____I__I_>I0______
____________I_<*1__I_<_____I__I_<*0______
_____________I>_I2__I>______I__I>_I1_____
_____________I_>I2__I_>_____I__I_>I1_____
_____________I_<*2__I__>____I__I_<*1_____
______________I>_I3__I__>____I__I>_I2____
______________I_>I3__I___>___I__I_>I2____
______________I_<*3__I____>__I__I_<*2____
_______________I>_I4__I____>__I__I>_I3___
_______________I_>I4__I_____>_I__I_>I3___
_______________I_<*4__I______>I__I_<*3___
________________I>_I5__I______>I__I>_I4__
________________I_>I5__I______<*__I_>I4__
________________I_<*5__I_____<_I__I_<*4__
            .
            .
            .

(b)
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Figure 3: Unsynchronized (a) and synchronized (b) ray clocks in moving reference frame.

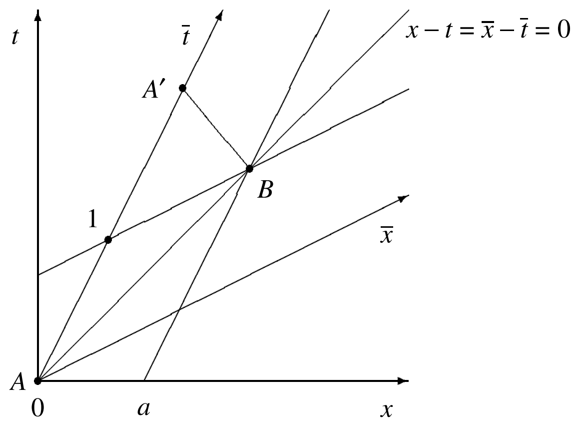

Figure 4: Construction of the coordinate axes $\overline{x}$ and $\overline{t}$ of a system which moves with constant velocity with respect to the medium.

2.4 Limits of covariance

One physical statement of relativity theory concerns symmetry. The relativity principle claims that all laws of physics have an identical form in all inertial frames. Despite attempts to construct fully Lorentz invariant CA models [4, 37, 29], this is certainly not true for many CA evolution rules.

It would have been not unreasonable to adopt Galilean conventions for the generation of space-time frames. In the same sense, it is not totally unreasonable to consider Galilean frames, in particular absolute space and time, in physics. This is not so much a question of principle but of practice. Whether it is more functional to define the ray speed to be constant in all space-time frames or to consider one preferred frame of reference, is a question of operationalizability and of compactness. For instance, a preferred frame of reference can be motivated on operational grounds, such as the redshift of the "cosmic background radiation," or by an explicit violation of the relativity principle. Yet, Occam's razor suggests that one uses symmetry whenever possible to obtain a theoretical description which is as compact as possible. This applies also to a Lorentz-invariant formulation of the virtual physics of CA, and of computer-generated universes in general.

Intrinsically, every CA cell corresponds to a "point," which has no extension at all, in whatever coordinate system. Since the time and space coordinates become dilatated, for systems moving with nonzero velocity with respect to the CA medium, there are "more CA cell points per unit length" packed into intrinsic space, as well as there are "more CA cycles per unit time interval" packed into intrinsic time. — The statement being a modern-day version of Zeno's paradox of the stadion (cf. [34], p. 41). This could give rise, at least in principle, to a breakdown of covariance.

3 Why no paradox?

Let us come back to Casti's sound creatures—assume they call themselves SMORONS—discover the possibility to generate and detect light; e.g., by sonoluminescence [10]. It can be expected that this discovery will cause a major trauma for the smorons, because they will find out that they can communicate much faster than by sound waves, on which their space-time frames are based. If they have applied the Einstein conventions for defining space-time frames, they would observe light as supersonic (i.e., faster-than-sound) signal, which propagates on space-like ($x^2 - t^2 > 0$) world lines. Such a phenomenon which might in principle be forward in time signalling in one inertial frame could be used for backward in time signalling in another inertial frame; there always exists some (orthochronous) Lorentz transformation L which transforms a space-like wordline with $t > 0$ into one with $t < 0$.

Given backward in time signalling, a classical time paradox [38, 22, 23, 24, 21, 2, 20, 13] can be formulated: Assume two observers A and B which can communicate *backward* in time; i.e., a signal

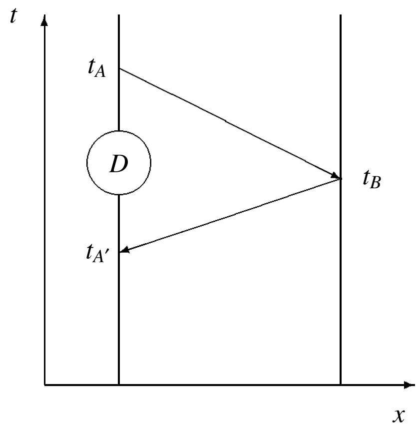

Figure 5: Configuration allowing a classical time paradox.

traverses the distance x_{AB} between them in time t_{AB} such that $t_{AB} < 0$. Then, the observer A might emit a signal at time t_A, which arrives the observer B in t_B, where it is reflected and is back at the observer A at a time $t_{A'} < t_A$, i.e., *before* observer A has emitted the original signal (Fig. 5). If one performs a "diagonalization," i.e., if one assumes that observer A emits a signal at time t_A if and only if *no* signal is absorbed at $t_{A'}$; observer A emits *no* signal at time t_A if and only if a signal is absorbed at $t_{A'}$, one ends up with the simplest form of time paradox.

The syntactic structure of this paradox closely resembles Cantor's diagonalization method (based on the ancient liar paradox), which has been applied by Gödel, Turing and others for undecidability proofs in a recursion theoretic setup [27, 19].

How could the smorons cope with such a time paradox? We discuss three related possibilities.

3.1 No free will due to cryptodeterminism

One could argue that sound consists of elementary constituents (such as atoms or molecules or clusters thereof), whose motion is ultimately governed by electromagnetic forces. Therefore, the valid theory is electromagnetism, and any theory "shell" ("level") such as the smoron theory of sound waves must ultimately be based upon (although not necessarily be totally derivable from) it. As has been pointed out earlier, such a "shell" is very similar to what Anderson [1] calls "emerging law" (cf. Schweber [28]). In this picture, the elementary constituents are a sort of "hidden parameters" for smoron physics; something they can neither observe nor control. Therefore, the smoron physics does not apply to configurations in which their physics "shell" is inappropriate. They are unable to control the events. This is why the theory is cryptodeterministic, and the smorons have no free will with respect to diagonalization—they will simply not be able to operationalize diagonalization purely in terms of sound waves.

3.2 Indeterminism

A related possibility for smoron physics is to allow superpositions of states represented by vectors of a linear space, preferably of a Hilbert space. Let the symbol $u \equiv (1, 0)$ stand for the state corresponding to a signal absorption, and $v \equiv (0, 1)$ stand for the state corresponding to *no* signal absorption, then the diagonalization process can be described by the `not`-operator $D \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. A consistent state is formed by the superposition

$$f := u + v \equiv (1, 1), \tag{5}$$

which is the fixed-point state of D. In smoron-physical terms, f corresponds to the impossibility to deterministically absorb or emit the pure states u, v.

3.3 Switching principle

A third, related, way of perceiving time paradoxa from faster-than-light signalling is based on the standard quantum field theoretic reinterpretation of negative energy particles moving forward in time as positive energy particles moving backward in time, which in turn are reinterpreted [30, 15] as positive energy *anti*particles moving forward in time with reversed charges and velocities [5, 14, 23]. In this process, any emission is reinterpreted as absorption and *vice versa.* This "switching principle" guarantees that faster-than-light effects cannot even in principle give rise to time paradoxes such as the above one. It is thereby assumed that one cannot control the re-interpreted emission, i.e., the corresponding absorption, thereby sacrificing free will.

4 What is this all good for?

One major goal of these considerations is the assertion that space and time are not God-given, metaphysical objects, but are subject to theoretical *construction*. The construction of space-time depends on *conventions*. Any inconsistency, for instance the possibility to construct time paradoxes, may be perceived as a problem of the improper, unfaithful construction of space and time rather than the impossibility to do certain tasks such as super-fast signalling. Any unfaithful construction of space-time may in turn be deeply rooted in the status quo of physical theory.

Appendix

A Transformation rules of light clock

$\varphi(>,\_,X) =>$, $\varphi(X,\_,<) =<$, $\varphi(\_,\_,\_) = \_$, $\varphi(X,\_,>) = \_$, $\varphi(<,\_,X) = \_$, $\varphi(\_,>,\_) = \_$, $\varphi(\_,<,\_) = \_$, $\varphi(\_,>,\mathtt{I}) =<$, $\varphi(\mathtt{I},<,\_) =>$, $\varphi(>,\mathtt{I},X) = *$, $\varphi(<,*,X) = \mathtt{I}$, $\varphi(X,<,*) = \_$, $\varphi(*,1,X) = 2$, $\varphi(*,2,X) = 3$, $\varphi(*,3,X) = 4$, $\varphi(*,4,X) = 5$, $\varphi(*,5,X) = 6$, $\varphi(*,6,X) = 7$, $\varphi(*,7,X) = 8$, $\varphi(*,8,X) = 9$, $\varphi(*,9,X) = 0$, $\varphi(*,0,X) = 1$, $\varphi(0,\_,X) = \_$, $\varphi(1,\_,X) = \_$, $\varphi(2,\_,X) = \_$, $\varphi(3,\_,X) = \_$, $\varphi(4,\_,X) = \_$, $\varphi(5,\_,X) = \_$, $\varphi(6,\_,X) = \_$, $\varphi(7,\_,X) = \_$, $\varphi(8,\_,X) = \_$, $\varphi(9,\_,X) = \_$, $\varphi(X,*,0) = *$, $\varphi(X,*,1) = *$, $\varphi(X,*,2) = *$, $\varphi(X,*,3) = *$, $\varphi(X,*,4) = *$, $\varphi(X,*,5) = *$, $\varphi(X,*,6) = *$, $\varphi(X,*,7) = *$, $\varphi(X,*,8) = *$, $\varphi(X,*,9) = *$, $\varphi(X,1,X) = 1$, $\varphi(X,2,X) = 2$, $\varphi(X,3,X) = 3$, $\varphi(X,4,X) = 4$, $\varphi(X,5,X) = 5$, $\varphi(X,6,X) = 6$, $\varphi(X,7,X) = 7$, $\varphi(X,8,X) = 8$, $\varphi(X,9,X) = 9$, $\varphi(X,0,X) = 0$, $\varphi(X,\mathtt{I},0) = \mathtt{I}$, $\varphi(X,\mathtt{I},1) = \mathtt{I}$, $\varphi(X,\mathtt{I},2) = \mathtt{I}$, $\varphi(X,\mathtt{I},3) = \mathtt{I}$, $\varphi(X,\mathtt{I},4) = \mathtt{I}$, $\varphi(X,\mathtt{I},5) = \mathtt{I}$, $\varphi(X,\mathtt{I},6) = \mathtt{I}$, $\varphi(X,\mathtt{I},7) = \mathtt{I}$, $\varphi(X,\mathtt{I},8) = \mathtt{I}$, $\varphi(X,\mathtt{I},9) = \mathtt{I}$, $\varphi(X,\mathtt{I},0) = \mathtt{I}$, $\varphi(X,\mathtt{I},X) = \mathtt{I}$, $\varphi(*,\_,X) = \_$, $\varphi(\_,\_,\mathtt{I}) = \_$, $\varphi(\mathtt{I},\_,\_) = \_$, $\varphi(\mathtt{I},>,\_) = \_$, $\varphi(\_,<,\mathtt{I}) = \_$.

X stands for any state.

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