Experimental realization of any discrete operator

K. Svozil
Institut für Theoretische Physik
University of Technology Vienna
Wiedner Hauptstraße 8-10/136
A-1040 Vienna, Austria
e-mail: svozil@tph.tuwien.ac.at
www: http://tph.tuwien.ac.at/~svozil
this file is at http://tph.tuwien.ac.at/~svozil/publ/every.tex

December 2, 1996

Abstract

A straightforward argument shows that, by allowing counterfactual elements of physical reality, any *arbitrary* discrete finite-dimensional operator corresponds to an observable.

As can be readily verified, any matrix A can be decomposed into two self-adjoint components A_1, A_2 as follows.

$$A = A_1 + iA_2 \tag{1}$$

$$A_1 = \frac{1}{2}(A + A^\dagger) =: \Re A, \tag{2}$$

$$A_2 = -\frac{i}{2}(A - A^\dagger) =: \Im A. \tag{3}$$

This is an extension of the decomposition of a complex number into its real and imaginary component; e.g., for any $a \in \mathbb{C}$, $a = a_1 + ia_2$, with $a_1 = (1/2)(a + a^*) =: \Re a \in \mathbb{R}$ and $a_2 = -(i/2)(a - a^*) =: \Im a \in \mathbb{R}$ if $A = a$ is a (1×1)-matrix. Since the trace is additive, this decomposition preserves the usual definition of quantum expectation values even for the case of non pure states.

Thus, if A_1 and A_2 would consistently be measurable, then A would consistently be measurable. As could be expected, in quantum mechanics, this is guaranteed only if their commutator vanishes; i.e.,

$$[A_1, A_2] = A_1A_2 - A_2A_1 = 0.$$

Note that, if $A = A^\dagger$ is self-adjoint, the decomposition is trivial; i.e., $A_1 = A$ and $A_2 = 0$.

Nevertheless, quantum mechanics could be extended to counterfactual elements of physical reality, as suggested by the Einstein-Podolski-Rosen (EPR) argument [3]. EPR state that "if, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." Thereby, EPR make no difference between an observable which is actually measured and one which could only be obtained by reasoning. The term *counterfactual* can be defined, in Max Jammer's words [5, pp. 9–10], as follows. "In general, an argument is called "counterfactual" if it involves a thought experiment the actual performance of which on a given system is made impossible because the conditions necessary for performing this experiment cannot be satisfied." A shorter definition is due to Roger Penrose. [9, p. 204], "... things that might have happened, although they did not happen." Surely enough, as has been shown by Bell [1] (with an explicit reference to Gleason's theorem [4]) and by Kochen and Specker [6], the naive assumption of non contextual counterfactuals gives rise to inconsistencies and therefore excludes a broad class of hidden parameter theories [10, 7].

With this *proviso*, one could nevertheless imagine Gedankenexperiments which "measure" an arbitrary discrete observable, irrespective of whether the corresponding operator is self-adjoint or not.—Here, the quotes surrounding "measure" mean that counterfactuals are involved. Assume, for instance, a spin one-half system representable by twodimensional Hilbert space. The matrix

$$A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

is not self-adjoint, but following equations (1)–(3) can be decomposed into two self-adjoint operators

$$A_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{1}{2}\sigma_1, \tag{4}$$

$$A_2 = \frac{1}{2}\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} = -\frac{1}{2}\sigma_2, \tag{5}$$

$$A = \sigma_1 - i\sigma_2. \tag{6}$$

Here, $\sigma_i, i = 1,2,3$ denote the Pauli spin matrices. In an EPR-type measurement setup drawn in Figure 1, two entangled spin one-half particles in the singlet state could be spatially separated and interrogated. Measurement on the first particle could yield the element of physical reality associated with σ_1; that is the spin state along the $x - axis$. Measurement on the second particle could yield the element of physical reality associated with σ_2; that is the spin state along the $y - axis$. These outcomes could then be combined according to Equation (1) to yield the element of physical reality associated with A.

It is not difficult to imagine that, since we can always decompose an arbitrary observable into just two self-adjoint observables, a generalized EPR-type experiment combined with the Reck–Zeilinger–Bernstein–Bertani setup [11], every discrete finite-dimensional operator which is not necessarily self-adjoint, is "measurable" (in the counterfactual sense).

References

[1] John S. Bell. On the problem of hidden variables in quantum mechanics. *Reviews of Modern Physics*, 38:447–452, 1966. Reprinted in [2, pp. 1-13].

[2] John S. Bell. *Speakable and Unspeakable in Quantum Mechanics.* Cambridge University Press, Cambridge, 1987.

[3] Albert Einstein, Boris Podolsky, and Nathan Rosen. Can quantum-mechanical description of physical reality be considered complete? *Physical Review*, 47:777–780, 1935. Reprinted in [13, pp. 138-141].

[4] Andrew M. Gleason. Measures on a closed subspaces of a Hilbert space. *Journal of Mathematics and Mechanics*, 6:885–893, 1957.

[5] Max Jammer. John Steward Bell and the debate on the significance of his contributions to the foundations of quantum mechanics. In A. van der Merwe, F. Selleri, and G. Tarozzi, editors, *Bell s Theorem and the Foundations of Modern Physics*, pages 1–23. World Scientific, Singapore, 1992.

[6] Simon Kochen and Ernst P. Specker. The problem of hidden variables in quantum mechanics. *Journal of Mathematics and Mechanics*, 17(1):59–87, 1967. Reprinted in [12, pp. 235–263].

[7] N. D. Mermin. Hidden variables and the two theorems of John Bell. *Reviews of Modern Physics*, 65:803–815, 1993.

[8] F. D. Murnaghan. *The Unitary and Rotation Groups.* Spartan Books, Washington, 1962.

[9] Roger Penrose. *Shadows of the Minds, A Search for the Missing Science of Consciouness.* Oxford University Press, Oxford, 1994.

[10] Asher Peres. *Quantum Theory: Concepts and Methods.* Kluwer Academic Publishers, Dordrecht, 1993.

[11] M. Reck, Anton Zeilinger, H. J. Bernstein, and P. Bertani. Experimental realization of any discrete unitary operator. *Physical Review Letters*, 73:58–61, 1994. See also [8].

[12] Ernst Specker. *Selecta.* Birkhäuser Verlag, Basel, 1990.

[13] John Archibald Wheeler and Wojciech Hubert Zurek. *Quantum Theory and Measurement.* Princeton University Press, Princeton, 1983.

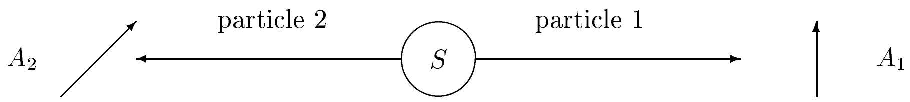

Figure 1: EPR-type setup for measurement of arbitrary operator decomposed into its self-adjoint real and imaginary part.