Quantum queries associated with equi-partitioning of states and multipartite relational encoding across space-time

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Abstract In the first part of this paper we analyze possible quantum computational capacities due to quantum queries associated with equi-partitions of pure orthogonal states. Special emphasis is given to the parity of product states and to functional parity. The second part is dedicated to a critical review of the relational encoding of multipartite states across (space-like separated) space-time regions; a property often referred to as "quantum nonlocality."

1 Unconventional properties for unconventional computing

At the heart of any unconventional form of computation (information processing) appears to be some (subjectively and "means relative" to the current canon of knowledge) strange, mind boggling, unexpected, stunning, surprising, hard to believe, feature or capacity of Nature. That is, in order to search for potentially unconventional information processing, we have to parse for empirical patterns and behaviour as well as for theoretical predictions which, relative to our expectations, go beyond our everyday "classical" experience of the world. "Unconventional" always is "means relative" and has to be seen in a historic context; that is, relative to our present means and capacities which we consider consolidated and conventional.

For the sake of some example, take the transmission of data from one point to another via satellite links or cables; or take (gps) navigation by time synchronization; or take the prediction of all sorts of phenomena, including weather or astronomical events. All these capacities appear conventional today, but would have been unconventional, or even magical, only 200 years ago.

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So what are the new frontiers? In what follows I shall mainly concentrate on some quantum physical capacities which are widely considered as potential resources for presently "unconventional" computation.

Before we begin the discussion, a *caveat* is in order. First, we should not get trapped by inappropriate yet convenient formal assumptions which have no operational consequences. For instance, all kinds of "infinity processes" have no direct empirical correspondence. In particular, classical continua abound in physics, but they need to be perceived rather as convenient though metaphysical "completion" of processes and entities which are limited by finite physical means.

We should also not get trapped by what Jaynes called *Mind Projection Fallacy* [1, 2], pointing out that *"we are all under an ego-driven temptation to project our private thoughts out onto the real world, by supposing that the creations of one's own imagination are real properties of Nature, or that one's own ignorance signifies some kind of indecision on the part of Nature."* Instead we should attempt to maintain a curiosity with *evenly-suspended attention* outlined by Freud [3] against *"temptations to project, what [the analyst] in dull self-perception recognizes as the peculiarities of his own personality, as generally valid theory into science."*

The postulate of "true," that is, ontological, randomness in Nature is such a fallacy, in both ways mentioned in the *caveat:* it assumes infinite physical resources (or maybe rather *ex nihilo* creation), as well as our capacity to somehow being able to "prove" this – a route blocked by recursion theory; in particular, by reduction to the Halting problem.

2 Quantum speedups by equi-decomposition of sets of orthogonal states

One of the mind boggling features of quantum information is that, unlike classical information, it can "reside," or be encoded into, the *relational properties* of multiple quanta [4, 5]. For instance, the singlet Bell state of, say, two electrons is defined by the following property (actually, two orthogonal spatial directions would suffice): if one measures the spin properties of these particles along some arbitrary spatial direction, then the spin value observed on one particle turns out to be always the negative spin value observed on the other particle – their relative spin value is negative – that is, either of "$+$" sign for the first particle and of "$-$" sign for the second one; or *vice versa*. The "rub," or rather compensation, for this fascinating encoding of information "across" particles appears to be that none of these individual particles has a definite individual spin value before the (joint) measurement. That is, all information encodable into them is exhausted by these relational specifications. This well recognized capacity of quantum mechanics could be conceived as the "essence of entanglement" [6].

Besides entanglement there is another capacity which is not directly related to relational properties of multipartite systems; yet it shares some similarities with the latter: the possibility to organize elementary, that is, binary (or, in general, d-ary) quantum queries resolving properties which can be encoded into (equi-)partitions of some set of pure states. If such partitions are feasible, then it is possible to obtain one bit (or, in general, dit) of information by staging such a single query *without* knowledge in what particular state the quantized system is.

From a different perspective any such (binary or d-ary) observable is related to a *partial* (i.e. incomplete) state identification [7, 8, 9]. Many of the fast quantum algorithms discussed in the literature depend on incomplete state identification.

Note that, in the binary case, any complete state identification – that is, setting up a complete set of quantum observables or queries capable to discriminate between and "locating" all single states – could be seen as the dual (observable) side of what can be considered an arbitrary state preparation for multipartite systems. This latter state preparation also features entanglement by allowing appropriate relational properties among the constituent quanta.

2.1 Parity of two-partite binary states

For the sake of a demonstration of the "unconventional" quantum speedup achievable through partial (incomplete) state identification, consider the four two-partite binary basis states $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$. Suppose we are interested in the even parity of these states. Then we could construct a *even parity operator* P *via* a spectral decomposition; that is,

$$\begin{aligned} \mathsf{P} &= 1 \cdot \mathsf{P}_- + 0 \cdot \mathsf{P}_+, \text{ with} \\ \mathsf{P}_- &= |01\rangle\langle 01| + |10\rangle\langle 10|, \\ \mathsf{P}_+ &= |00\rangle\langle 00| + |11\rangle\langle 11|, \end{aligned} \tag{1}$$

which yields even parity "0" on $|00\rangle$ as well as $|11\rangle$, and even parity "1" on $|01\rangle$ as well as $|10\rangle$, respectively. Note that P_- as well as P_+ are projection operators, since they are idempotent; that is, $\mathsf{P}_-^2 = \mathsf{P}_-$ and $\mathsf{P}_+^2 = \mathsf{P}_+$.

Thereby, the basis of the two-partite binary states has been effectively equipartitioned into two groups of even parity "0" and "1;" that is,

$$\big\{\{|00\rangle, |11\rangle\}, \{|01\rangle, |10\rangle\}\big\}. \tag{2}$$

The states associated with the propositions corresponding to the projection operators P_- for even parity one and P_+ for even parity zero of the two bits are entangled; that is, this information is only expressed in terms of a *relational property* – in this case parity – of the two quanta between each other [4, 5].

2.2 Parity of multi-partite binary states

This equi-partinioning strategy [7, 10] to determine parity with a single query can be generalized to determine the parity of multi-partite binary states. Take, for example, the even parity of three-partite binary states definable by

$$\begin{aligned} \mathbf{P} &= 1\cdot\mathbf{P}_- + 0\cdot\mathbf{P}_+, \text{ with} \\ \mathbf{P}_- &= |001\rangle\langle 001| + |010\rangle\langle 010| + |100\rangle\langle 100| + |111\rangle\langle 111|, \\ \mathbf{P}_+ &= |000\rangle\langle 000| + |011\rangle\langle 011| + |101\rangle\langle 101| + |110\rangle\langle 110|. \end{aligned} \tag{3}$$

Again, the states associated with the propositions corresponding to the projection operators $\mathbf{P}_-$ for even parity one and $\mathbf{P}_+$ for even parity zero of the three bits are entangled. The basis of the three-partite binary states has been equi-partitioned into two groups of even parity "0" and "1;" that is,

$$\begin{aligned} \big\{ &\{|000\rangle, |011\rangle, |101\rangle, |110\rangle\}, \\ &\{|001\rangle, |010\rangle, |100\rangle, |111\rangle\}\big\}. \end{aligned} \tag{4}$$

3 Parity of Boolean functions

It is well known that Deutsch's problem – to find out whether the output of a binary function of one bit is constant or not; that is, whether the two outputs have even parity zero or one – can be solved with one quantum query [11, 12]. Therefore it might not appear totally unreasonable to speculate that the parity of some Boolean function – a binary function of an arbitrary number of bits – can be determined by a single quantum query. Even though we know that the answer is negative [13] it is interesting to analyze the reason why this parity problem is "difficult" even for quantum resources, in particular, quantum parallelism. Because an answer to this question might provide us with insights about the (in)capacities of quantum computations in general.

Suppose we define the functional parity $P(f_i)$ of an n-ary function $f_i = (g_i+1)/2$ *via* a function $g_i(x_1,\ldots,x_n) \in \{-1,+1\}$ and

$$P(g_i) = \prod_{x_1,\ldots,x_n\in\{0,1\}} g_i(x_1,\ldots,x_n). \tag{5}$$

Let us, for the sake of a direct approach of functional parity, consider all the 2^{2^n} Boolean functions $f_i(x_1,\ldots,x_n)$, $0 \le i \le 2^{2^n}-1$ of n bits, and suppose that we can represent them by the standard quantum oracle

$$\begin{aligned} U_i(|x_1,\ldots,x_n\rangle|y\rangle) &= \\ |x_1,\ldots,x_n\rangle|y \oplus f_i(x_1,\ldots,x_n)\rangle \end{aligned} \tag{6}$$

as a means to cope with possible irreversibilities of the functions f_i. Because $f_i \oplus f_i = 0$, we obtain $U_i^2 = \mathbf{I}$ and thus reversibility of the quantum oracle. Note that all the resulting $n+1$-dimensional vectors are not necessarily mutually orthogonal.

For each particular $0 \le i \le 2^{2^n} - 1$, we can consider the set

$$F_i = \{f_i(0,\dots,0),\dots,f_i(1,\dots,1)\} \tag{7}$$

of all the values of f_i as a function of all the 2^n arguments. The set

$$\begin{aligned} V &= \{F_i \mid 0 \le i \le 2^{2^n} - 1\} \\ &= \left\{\{f_i(0,\dots,0),\dots,f_i(1,\dots,1)\} \mid 0 \le i \le 2^{2^n} - 1\right\} \end{aligned} \tag{8}$$

is formed by all the 2^{2^n+n} Boolean functional values $f_i(x_1,\dots,x_n)$. Moreover, for every one of the 2^{2^n} different Boolean functions of n bits the 2^n functional output values characterize the behavior of this function completely.

In the next step, suppose we equi-partition the set of all these functions into two groups: those with even parity "0" and "1," respectively. The question now is this: can we somehow construct or find two mutually orthogonal subspaces (orthogonal projection operators) such that all the parity "0" functions are represented in one subspace, and all the parity "1" are in the other, orthogonal one? Because if this would be the case, then the corresponding (equi-)partition of basis vectors spanning those two subspaces could be coded into a quantum query [7] yielding the parity of f_i in a single step.

We conjecture that involvement of one or more additional auxiliary bits (e.g., to restore reversibility for nonreversible f_i's) cannot improve the situation, as any uniform (over all the functions f_i) and non-adaptive procedure will not be able to generate proper orthogonality relations.

We know that for $n = 1$ this task is feasible, since (we re-coded the functional value "0" to "-1")

$$\begin{array}{c|ccc} f_i & P(f_i) & f_i(0) & f_i(1) \\ \hline f_0 & 0 & -1 & -1 \\ f_1 & 0 & +1 & +1 \\ f_2 & 1 & -1 & +1 \\ f_3 & 1 & +1 & -1 \end{array} \tag{9}$$

and the two parity cases "0" and "1," are coded into orthogonal subspaces spanned by $(1,1)$ and $(-1,1)$, respectively.

This is no longer true for $n = 2$; due to an overabundance of functions, the vectors corresponding to both parity cases "0" and "1" span the entire Hilbert space:

f_i	$P(f_i)$	$f_i(00)$	$f_i(01)$	$f_i(10)$	$f_i(11)$
f_0	0	-1	-1	-1	-1
f_1	0	-1	-1	$+1$	$+1$
f_2	0	-1	$+1$	-1	$+1$
f_3	0	-1	$+1$	$+1$	-1
f_4	0	$+1$	-1	-1	$+1$
f_5	0	$+1$	-1	$+1$	-1
f_6	0	$+1$	$+1$	-1	-1
f_7	0	$+1$	$+1$	$+1$	$+1$
f_8	1	-1	-1	-1	$+1$
f_9	1	-1	-1	$+1$	-1
f_{10}	1	-1	$+1$	-1	-1
f_{11}	1	-1	$+1$	$+1$	$+1$
f_{12}	1	$+1$	-1	-1	-1
f_{13}	1	$+1$	-1	$+1$	$+1$
f_{14}	1	$+1$	$+1$	-1	$+1$
f_{15}	1	$+1$	$+1$	$+1$	-1

(10)

3.1 Proper specification of state discrimination

The results of this section are also relevant for making precise Zeilinger's *foundational principle* [4, 5] claiming that an n-partite system can be specified by exactly n bits (dits in general). The issue is what exactly is a "specification?"

We propose to consider a specification appropriate if it can yield to an equipartitioning of all pure states of the respective quantized system. That is, to give an example, the parity of states could serve as a proper specification, but functional parity in general (for more that two quanta) cannot.

4 Relativity theory *versus* quantum inseparability

Let us turn our attention to another "unconventional" quantum resource, which is mostly encountered at (but not restricted to) spatially separated entangled states: the so-called "quantum nonlocality;" and, in particular, on the paradigm shift of our perception of physical space and time.

First, let us keep in mind that, in the historic perspective it is quite evident why our current theory of space-time, relativity theory [14, 15], does not directly refer to quanta: it was created when quantum mechanics was "unborn," or at least in its early infancy. Indeed, in 1905 it was hardly foreseeable that Planck's self-denominated [16, p. 31] *"Akt der Verzweiflung"* ("act of despair") – committed five years ago in 1900 for the sake of theoretically accommodating precision measurements of the blackbody radiation – would be extended into one of the most powerful physical theories imagined so far. Therefore it should come as no surprise that all operationalizations and conventions implemented by relativity theory, in particular, simultaneity, refer to classical, pre-quantum, physics.

Besides its applicability and stunning predictions and consequences (such as, for instance "$E = mc^2$," as well as the unification of classical electric and magnetic phenomena) the triumph of special relativity resides in its structural as well as formal clarity: by adopting certain conventions (which were essentially adopted from railroad traffic [17, 18] and are also used by *Cristian's Algorithm* for data network synchronization), and by fixing the speed of electromagnetic radiation for all reference frames (together with the requirement of bijectivity), the Lorentz transformations result from theorems of incidence geometry [19, 20]. Beyond formal conventions, the physical content resides in the form invariance of the equations of motion under such transformations.

In view of these sweeping successes of classical relativity theory it might not be surprising that Einstein, one of the creators of quantum mechanics, never seriously considered the necessity to adapt the concepts of space-time to the new quantum physics. On the contrary – Einstein seemed to have prioritized relativity over quantum theory; the latter one he critically referred to as [21, p. 113] *"noch nicht der wahre Jakob" ("not yet the true [final] answer").* Time and again Einstein came up with predictions of quantum mechanics which allegedly discredited the (final) validity of quantum theory.

In a letter to Schrödinger dated June 19th, 1935 [22, 23] Einstein concretized and clarified his uneasiness with quantum theory previously published in a paper with Podolsky and Rosen [24] (*"written by Podolsky after many discussions"* [22]). In this communication Einstein insisted that the wave function of a subsystem A of (entangled) particles cannot depend on whatever measurements are performed on its spatially separated (i.e. separated by a space-like interval) "twin" subsystem B: in his own (translated from German [1]) words: *"The true state of B cannot depend on which measurement I perform on A."* Pointedly stated, the *"separability principle" asserts that any two spatially separated systems possess their own separate real state* [23].

The separability principle is not satisfied for entangled states [25, 26]; in particular, if general two-partite state

$$|\Psi\rangle = \sum_{i,j\in\{-,+\}} \alpha_{ij}|ij\rangle, \text{ with } \sum_{i,j\in\{-,+\}} |\alpha_{ij}|^2 = 1$$

does not satisfy factorizability [12, p. 18] requiring $\alpha_{--}\alpha_{++} = \alpha_{+-}\alpha_{-+}$. That is, if $\alpha_{--}\alpha_{++} \neq \alpha_{+-}\alpha_{-+}$, then $|\Psi\rangle$ cannot be factored into products of single particle states.

Even in his later years Einstein was inclined to take relativistic space-time as the primary framework; thereby prioritizing it over fundamental quantum mechanical inseparability; in particular, when it comes to multipartite situations [27, 23].

[1] Einstein's (underlined) original German text: *"Der wirkliche Zustand von B kann nicht davon abhängen, was für eine Messung ich an A vornehme."*

5 Proximity and apartness in quantum mechanics

In what follows we propose that, when it comes to microphysical situations, in particular, when entanglement is involved, the provenance of classical relativity theory over quantum mechanics has to be turned upside down: while entangled quanta may epistemically (and for many practical purposes [28]) appear "separated," or "apart," or "distinct" to a classical observer ignorant of their relational properties (cf. earlier discussion in Section 2) encoded "across these quanta," quantum mechanically they are treated holistically "as one."

The pretension of any such observer, or the possibility to actually perceive entangled quanta as being "spatially separated" (by disregarding their correlations) should not be seen as a principal property, but rather as a *"means relative"* one.

For the sake of an example, take the two-particle singlet Bell state $|\Psi^-\rangle = (1/\sqrt{2})(|+-\rangle - |-+\rangle)$, which, by identifying $|-\rangle \equiv (0,1)$ and $|+\rangle \equiv (1,0)$, can be identified with the four-dimensional vector whose components in tuple form are $|\Psi^-\rangle \equiv (1/\sqrt{2})[(1,0)\otimes(0,1)-(0,1)\otimes(1,0)] = (1/\sqrt{2})(0,1,-1,0)$. The separability principle is not satisfied, since $0\cdot 0 \neq 1\cdot(-1)$. So, from the point of view of those entangled state observables, the quanta appear inseparable.

And yet, the same quanta can be perfectly localized and distinguished by resolving them spatially. This situation – the occurrence of both inseparability and (spatial) distinguishability – has caused a lot of confusion. This is particularly serious if one of these distinct viewpoints on the quantized system, say, spatial separability and locatedness of the particles, is meshed with the inseparability of the spin observables when the latter ones are relationally defined. An yet, we might envision that, with this dual situation we could get a handle on quantum inseparability (*via* encoding of relational information) by spatially separated detectability of the quanta forming this entangled state. Alas this is impossible, because the relational properties do not reveal themselves by individual outcomes – only when all these (relational) outcomes are considered together do the relational properties reveal themselves.

Of course, this would be totally different if it would be possible to wilfully *force* any particular handle or side or component of the entangled state, thereby effectively forcing the respective (relational property on the other handle or side or component. So far, despite speculative attempts to utilize stimulated emission [29], there is no indication that this might be physically feasible.

6 Summary

The first part of this article has been dedicated to quantum queries relating to properties which can be encoded in terms of (equi-)partitioning of states. We have been particularly interested in the parity of products of binary states, and also in the parity of Boolean functions; that is, dichotomic functions of bits. Thereby we have presented criteria for the (non-) existence of quantum oracles.

In the second part of this article we have argued that, instead of perceiving entangled quanta in an *a priori* "space-time theater," space-time is a secondary, derived concept of our mind which needs to be operationally constructed by conventions and observations. This is particularly true for multipartite entangled states, and their spatio-temporal interconnectedness. Such an approach leaves no room for any hypothetical inconsistency in quantum space-time, and no mind-boggling "peaceful coexistence" with relativity theory.

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