Contextuality, uniqueness property & quantum computing

http://tph.tuwien.ac.at/∼svozil/publ/2005-qupon-pres.pdf

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May 2005

(Incomplete) history

K.S., JPA, in press; quant-ph/0401113; K.S., AIP Conference Proceedings 750,quant-ph/0406014

- Bohr 1949 mentioned "the impossibility of any sharp separation between the behavior of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear."
- **Bell 1966 stated that the "... result of an observation may reasonably depend not only on the state of the system ... but also on the complete disposition of the apparatus."**
- Kochen and Specker and related theorems (Kamber, Alda, Zierler&Schlesinger, Bell-type, ...) express the "scarcity" or even nonexistence of (dispersion free) two-valued states interpretable as classical truth assignments

Last resort of realism?

- Realism: "some entities sometimes exist without being experienced by any finite mind"
- Idealism: "we have not the faintest reason for believing in the existence of unexperienced entities. [[Realism]] has been adopted ... solely because it simplifies our view of the universe."
- Counterfactual events: "events which would have occurred if something had happened which did not happen"
- **Classical principle of omni-measurability: all classical observables are defined; so should be all quantum observables.**
- **"Solution:" In view of KS, GHZ, ... quantum omni-measurability can be maintained at the price of contextuality. This amounts to a modified realism.**

Simplest example: EPR-type setup

A *context* is defined as a single (nondegenerate) "maximal" self-adjoint operator. Take five observables A, B, C, D, K with two systems of *contexts* $\{A, B, C\}$ and $\{D, K, A\}$, which are interconnected by A.

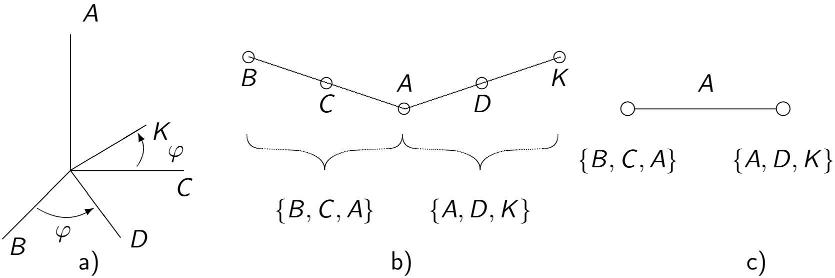
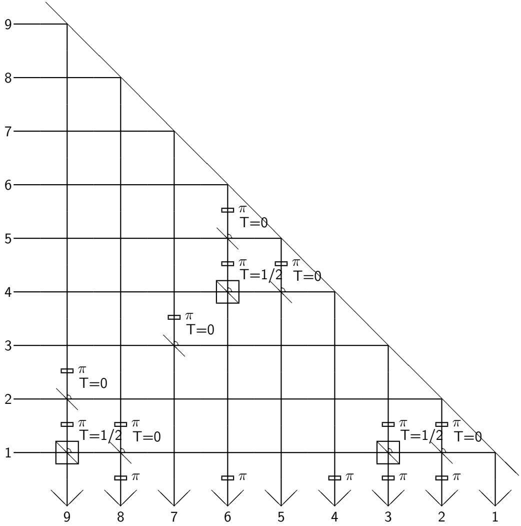
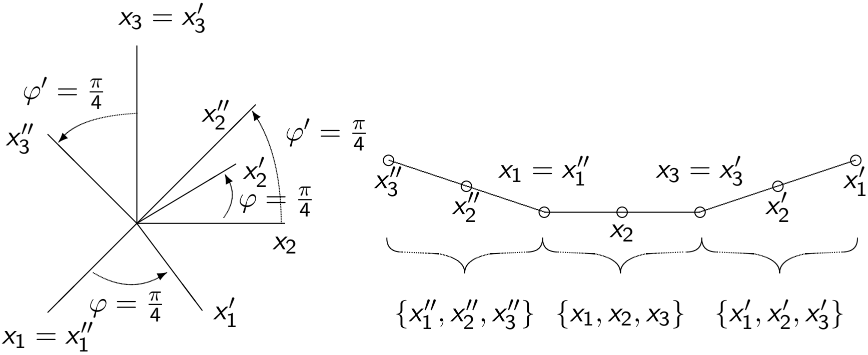


Figure: a) Two tripods with a common leg; b) Greechie (orthogonality) diagram; c) Tkadlec diagram.

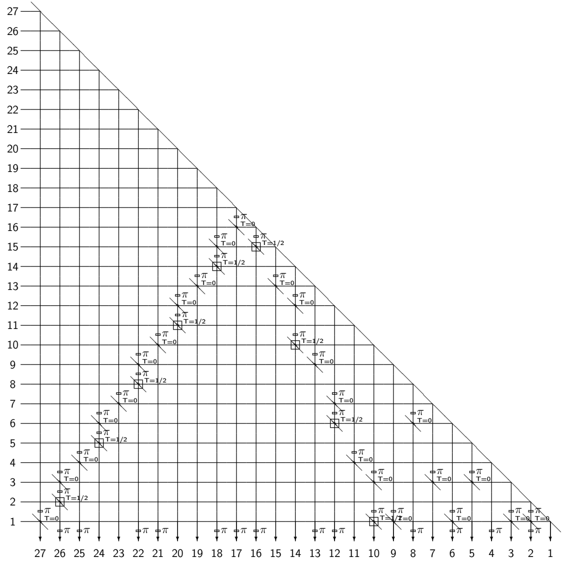
Interferometric realization



Three three-state particles



Interferometric implementation



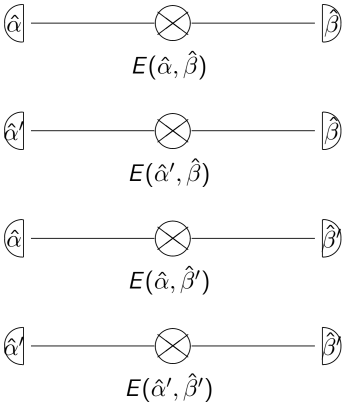
No empirical evidence of contextuality

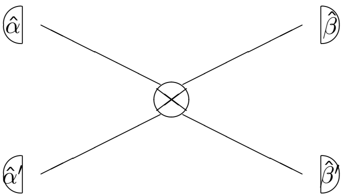
- In cases where contextuality could be tested, it is absent.
- In other cases (KS, GHZ, ...) where it is claimed, it cannot be tested.
- Speculative proposal I: abandonment of omni-measurability (why assume more than measurable?)
- Speculative proposal II: context translation principle—in case of context mismatch between preparation & measurement, measurement apparatus introduced stochastic noise during context translation.

"Explosion views"

quant-ph/0206076

In "simultaneous" measurements, all single measurements are pairwise spatially separated and temporally coincide in some reference frame.





$E(\hat\alpha,\hat\beta)$, $E(\hat\alpha',\hat\beta)$, $E(\hat\alpha,\hat\beta')$, $E(\hat\alpha',\hat\beta')$

Uniqueness property

- k-partite states have the *uniqueness property* if knowledge of a property of one particle entails the certainty that, if this property were measured on the other three particles as well, the outcome of the measurement would be a unique function of the outcome of the measurement actually performed.
- Protection scheme: from $k = 3$ onwards, no state has the uniqueness property in a sufficiently number of spatial directions to facilitate simultaneous Bell (KS, GHZ, ...) measurements. (Otherwise complete contradiction.)
- Example: take the two four spin–1/2 singlet states

$$
\begin{aligned}
|\Psi_{4,s_1}\rangle &= (|\Psi_{2,s}\rangle)^2 = \frac{1}{2}(|+-\rangle - |-+\rangle)(|+-\rangle - |-+\rangle), \\
|\Psi_{4,s_2}\rangle &= \frac{1}{\sqrt{3}}\Big[|++--\rangle + |--++\rangle \\
&\qquad -\frac{1}{2}(|+-\rangle + |-+\rangle)(|+-\rangle + |-+\rangle)\Big].
\end{aligned}
$$

Spread of information “across” single quanta in multipartite states

quant-ph/0505129

- One advantage of quantum algorithms over classical computation is the possibility to spread out, process and extract information in multipartite configurations in coherent superpositions of classical states.
- Quantum state identification problems based on a proper partitioning of mutually orthogonal sets of states.
- Question: Is it possible to encode equibalanced decision problems (e.g., Deutsch’s problem and Parity) into quantum systems, so that a single invocation of a filter used for state discrimination suffices to obtain the result?

Identifying states among contexts

The general problem to (uniquely) identify states among contexts resulting from k particles in $n = 2$ or more dimensions per particle can be solved [N. Donath and K. Svozil, PRA 65, 044302 (2002). quant-ph/0105046; K. Svozil, PRA 66, 044306 (2002). quant-ph/0205031;] *via* a system of k co-measurable filters $\mathbf{F}_i$, $i = 1, \ldots, k$ with the following properties:

- Every filter $\mathbf{F}_i$ corresponds to an operator (or a set of operators) which generate an equi–n-partition of the d-dimensional state space into k slices (i.e., partition elements) containing $d/n = d^{1-1/k}$ states per slice. A filter is said to separate two eigenstates if the eigenvalues are different.
- From each one of these k partitions, take an arbitrary element. The intersection of the elements of all different partitions results in a *single* one of the $d = n^k$ different states.
- The union of all those single states generated by the intersections is the entire set of states.

Summary

- Analysis of quantum computations in terms of state identification whose complexity grows linearly with the number of bits.
- Characterization this domain by partitions of state space, as well as by unitary transformations of the associated filter systems.
- Deutsch's problem OK, Parity not OK, Recursion & interation (not OK?)

Thank you for your attention!