

## Are simultaneous Bell measurements possible?

**Karl Svozil**

Institute for Theoretical Physics, University of Technology Vienna,  
Wiedner Hauptstrasse 8-10/136, A-1040 Vienna, Austria  
E-mail: [svozil@tuwien.ac.at](mailto:svozil@tuwien.ac.at)

*New Journal of Physics* **8** (2006) 39

Received 2 February 2006

Published 10 March 2006

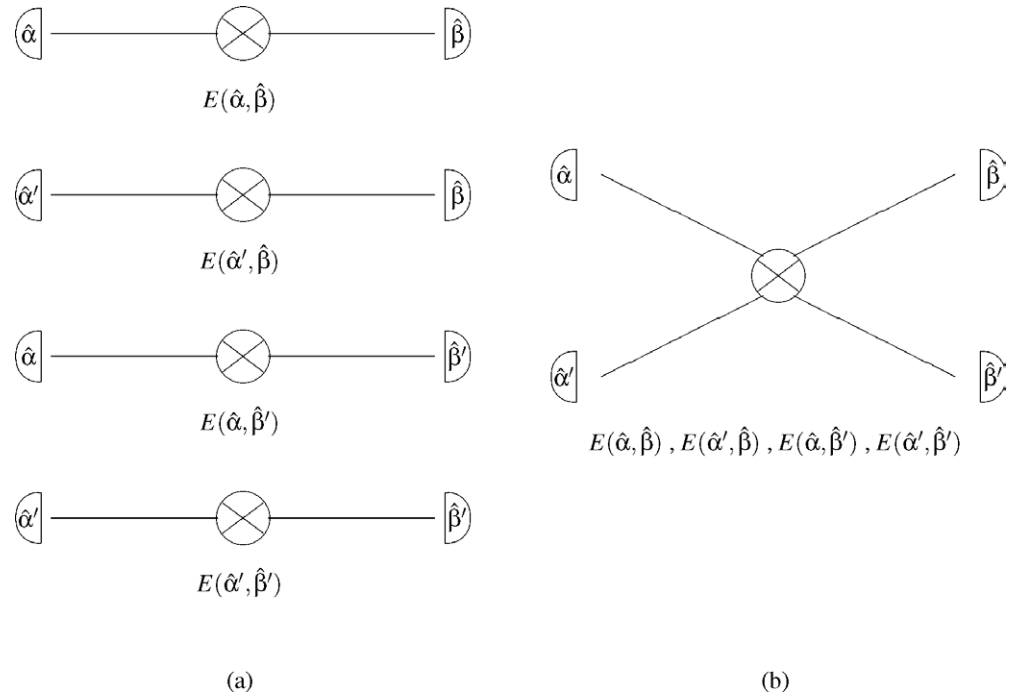
Online at <http://www.njp.org/>

doi:10.1088/1367-2630/8/3/039

**Abstract.** All experimental tests of Bell-type inequalities and Greenberger–Horne–Zeilinger setups rely on the separate and successive measurement of the terms involved. We discuss possibilities of experimental setups to measure all relevant terms simultaneously in a single experiment and find this to be impossible. One reason is the lack of multi-partite states which are unique in the sense that a measurement of some observable on one particle fixes the value of the corresponding observables of the other particles as well.

One motivation for the beautiful Bell-type experiment by Weihs *et al* [1] has been concerns [2] about the implicit coincidence between photon flight time and the specific switching frequency chosen in one of the first experiments [3] to test Bell-type inequalities. As pointed out by Gill *et al* [4]–[6] and by Larsson and Gill [7], any effectively computable synchronization strategy (e.g., [8]) fails for space-like separated observers who choose to switch the measurement directions at random [9, 10]. For Greenberger–Horne–Zeilinger (GHZ) measurements, the issue has already been discussed in the original paper by Pan *et al* [11]. Yet, there might remain reservations and uneasiness related to the fact that in all the experiments performed so far, different terms in the Bell-type inequalities have been measured consecutively, one after another, in different experiment setups.

In what follows we shall investigate, as a second and arguably conceptually more gratifying alternative, the feasibility to either measure or counterfactually infer all required entities simultaneously. By ‘simultaneous’ measurement we mean that all single measurements are pairwise spatially separated and temporally coincide in some reference frame (presumably in the centre-of-mass frame of the particles involved). Note also that, due to the apparent randomness and parameter independence of the single outcomes in the correlation experiment, relativistic locality is possible.



**Figure 1.** (a) Terms entering the CHSH inequality are usually measured by four different experiments on two-partite singlet states; (b) Scheme for the simultaneous measurement of all expectations values for the four directions  $\hat{a}$ ,  $\hat{a}'$ ,  $\hat{b}$  and  $\hat{b}'$  entering the CHSH inequality in a single experiment, requiring a four-partite state with the uniqueness property.

Take, for example, the Clauser–Horne–Shimony–Holt (CHSH) inequality  $|E(\hat{a}, \hat{b}) + E(\hat{a}, \hat{b}') + E(\hat{a}', \hat{b}) - E(\hat{a}', \hat{b}')| \leq 2$  containing the four terms  $E(\hat{a}, \hat{b})$ ,  $E(\hat{a}, \hat{b}')$ ,  $E(\hat{a}', \hat{b})$  and  $E(\hat{a}', \hat{b}')$  associated with the four measurement directions  $\hat{a}$ ,  $\hat{a}'$ ,  $\hat{b}$  and  $\hat{b}'$ , respectively. Since in the usual Einstein–Podolsky–Rosen (EPR) type of setup only two particles occur, merely two of the four directions can be measured and counterfactually inferred in every experimental run. As depicted in figure 1(a), four experimental runs with different parameter settings  $\hat{a}-\hat{b}$ ,  $\hat{a}-\hat{b}'$ ,  $\hat{a}'-\hat{b}$  and  $\hat{a}'-\hat{b}'$  are necessary to collect the frequency counts necessary for the CHSH inequality.

In contrast to this consecutive measurements, figure 1(b) depicts the hypothetical configuration associated with simultaneous measurements all the four parameters  $\hat{a}$ ,  $\hat{a}'$ ,  $\hat{b}$  and  $\hat{b}'$  in a *single* experimental run; i.e., without the necessity to change the setup for measuring different joint probabilities.

Simultaneity presents a big conceptual challenge, both theoretically and experimentally. While entangled multi-partite (singlet) states abound, the EPR argument [12] requires the unambiguous existence of (counterfactual) elements of physical reality. Theoretically, this means that we have to find four-partite states with the *uniqueness property* [13] such that knowledge of a property of one particle entails the certainty that, if this property were measured on the other three particles as well, the outcome of the measurement would be a unique function of the outcome of the measurement actually performed. More than that, such a uniqueness property must hold for all the observables associated with the four measurement directions  $\hat{a}$ ,  $\hat{a}'$ ,  $\hat{b}$  and  $\hat{b}'$ ,

respectively. The uniqueness property differs from elements of physical reality in that the latter ones, in order to be definable by EPR type experimental setups, require the uniqueness property. However, absence of the uniqueness property does not necessarily imply that no elements of physical reality exists, as certain elements of physical reality might not be measurable via EPR type experimental setups. The existence of elements of physical reality is a necessary but not a sufficient condition for uniqueness of multi-partite quantum states.

For more than two quanta, no state satisfying the uniqueness property has ever been proposed. The generalized four-partite GHZ–Mermin state, for example, has the uniqueness property only for a *single* measurement direction, in which it takes a form containing only two terms  $|\Psi\rangle = (1/\sqrt{2})(|++++\rangle + |-- --\rangle)$ ; in all other directions,  $|\Psi\rangle$  is a superposition of  $2^4$  terms, which are by far too numerous for a uniqueness property requiring just two terms for at least four measurement directions. Natural candidates would be singlet states, which are form invariant with respect to changes of the measurement directions. Alas, group theoretical methods reveal that no singlet state of four or more particles (in arbitrary dimensions of Hilbert space greater than one) exists which satisfies the uniqueness property required for simultaneous measurements of more than two properties. (States representable by only a single term in a particular measurement direction, such as  $|++++\rangle$ , have  $2^4$  terms in all the other directions and are non-unique for those directions.)

Consider, for example, two-state particles; their states being labelled by ‘+’ and ‘–’, respectively. The only two-partite singlet state is the usual Bell state  $|\Psi_{2,2,s}\rangle = (1/\sqrt{2})(|+-\rangle - |-+\rangle)$ . There exist two four-partite singlet states (and none for three particles), namely

$$|\Psi_{2,4,s_1}\rangle = \frac{1}{2}(|+-\rangle - |-+\rangle)(|+-\rangle - |-+\rangle), \quad (1)$$

$$|\Psi_{2,4,s_2}\rangle = \frac{1}{\sqrt{3}}[|++--\rangle + |--++\rangle - \frac{1}{2}(|+-\rangle + |-+\rangle)(|+-\rangle + |-+\rangle)]. \quad (2)$$

Both of these four-partite states are non-unique in all directions, for detection of, say, state ‘–’ on the first particle leaves open the possibility to find the third and fourth particles either in states ‘+’ or in ‘–’ for  $|\Psi_{2,4,s_1}\rangle$ . Likewise, the proposition

*‘The first particle has spin state ‘–’.’*

does not fix a single term of  $|\Psi_{2,4,s_2}\rangle$  in equation (2), but rather leaves open one of the two possibilities

*‘The second (third, fourth) particle has spin state ‘–’;’*

*‘The second (third, fourth) particle has spin state ‘+’;’*

both occurring at random.

This ambiguity gets worse as the number of particles increases. The  $2n$ -partite singlet states (singlet states with an odd number of particles do not exist) with the least number of terms are

$$|\Psi_{2,2n,s_1}\rangle = \frac{1}{2^{n/2}}(|+-\rangle - |-+\rangle)^n. \quad (3)$$

For  $n > 1$ , they are non-unique.

Also for singlet states of three-state particles, the uniqueness limit is reached for two particles; i.e., for (‘ $\pm$ ’ means here ‘ $\pm 1$ ’)  $|\Psi_{3,2,s}\rangle = (1/\sqrt{3})(|+-\rangle + |-+\rangle - |00\rangle)$ . The only singlet state of three spin-one particles is

$$|\Psi_{3,3,s}\rangle = \frac{1}{\sqrt{6}} (|-+0\rangle - |-0+\rangle + |+0-\rangle - |+ -0\rangle + |0 - +\rangle - |0 + -\rangle), \quad (4)$$

which does not allow unambiguous counterfactual reasoning. The situation gets worse for singlet states of four or more spin one quanta, as all of the three singlet states  $|\Psi_{3,4,s_1}\rangle$ ,  $|\Psi_{3,4,s_2}\rangle$ ,  $|\Psi_{3,4,s_3}\rangle$  of four spin-one quanta

$$\begin{aligned} |\Psi_{3,4,s_1}\rangle = \frac{1}{\sqrt{5}} [ & \frac{2}{3} |0000\rangle + |- - + +\rangle + | + + - -\rangle - \frac{1}{2} (|- 00+\rangle + |0 - 0+\rangle + |- 0 + 0\rangle \\ & + |0 - +0\rangle + |0 + -0\rangle + |+ 0 - 0\rangle + |0 + 0 -\rangle + | + 00 -\rangle) + \frac{1}{3} (|00 - +\rangle \\ & + |- +00\rangle + | + -00\rangle + |00 + -\rangle) + \frac{1}{6} (| - + - +\rangle + | + - - +\rangle \\ & + | - + + -\rangle + | + - + -\rangle)], \end{aligned} \quad (5)$$

$$\begin{aligned} |\Psi_{3,4,s_2}\rangle = \frac{1}{2\sqrt{3}} ( & |- 00+\rangle - |0 - 0+\rangle - |0 + 0-\rangle + |+ 00-\rangle - |- 0 + 0\rangle + |0 - +0\rangle + |0 + -0\rangle \\ & - |+ 0 - 0\rangle + |- + + -\rangle - |+ - + -\rangle - |- + - +\rangle + |+ - - +\rangle), \end{aligned} \quad (6)$$

$$|\Psi_{3,2n,s_3}\rangle = \frac{1}{3^{n/2}} (|+-\rangle + |-+\rangle - |00\rangle)^n \quad (7)$$

are non-unique. From all singlet states of  $2n$  spin-one particles,  $|\Psi_{3,2n,s_3}\rangle$  contains the least number of terms. For  $n > 1$ , they are non-unique. In general, for an even number of  $2n$  spin- $j$  particles, the singlet states containing the least number of terms can be written as  $|\Psi_{j,2n,s}\rangle = (|\Psi_{j,2,s}\rangle)^n$ , where  $|\Psi_{j,2,s}\rangle$  is the two-partite singlet state.

Classically, simultaneous measurements can be performed on an arbitrary number of particles and for arbitrary many observables. They are not bound by the quantum no-cloning theorem and complementarity.

Indeed, quantum probabilities do not allow the consistent co-existence of classical truth tables already for the usual Bell-type configurations such as CHSH [14, 15]. We therefore conclude that it is impossible to construct quantum states of four or more particles with the uniqueness property for four or more directions. Likewise, because of non-uniqueness, the observables involved in a Kochen–Specker-type argument cannot be measured simultaneously.

Let us briefly summarize as follows. The proposed scheme is not different from the standard treatment of EPR-type setups, since elements of physical reality are not directly measured but counterfactually inferred. One advantage of simultaneous measurements and inference might be the direct evaluation of all quantities involved; as compared to a serial, consecutive approach which is usually adopted. However, as the EPR argument is based on the uniqueness of the counterfactual properties, any such method fails already for more than two entangled particles.

A new kind of protection scheme seems to emerge which, due to the impossibility to find states with the required uniqueness property, forbids more direct experiments to measure all terms in Kochen–Specker-type, Bell-type or GHZ-type configurations at once. It is straightforward to deduce non-uniqueness for singlet states of three or more particles. An indirect argument, based on Bell- and Kochen–Specker-type theorems, proves the non-existence of general states with the uniqueness property.

## References

- [1] Weihs G, Jennewein T, Simon C, Weinfurter H and Zeilinger A 1998 Violation of Bell's inequality under strict Einstein locality conditions *Phys. Rev. Lett.* **81** 5039–43 <http://dx.doi.org/10.1103/PhysRevLett.81.5039>
- [2] Zeilinger A 1986 Testing Bell's inequalities with periodic switching *Phys. Lett. A* **118** 1–2 [http://dx.doi.org/10.1016/0375-9601\(86\)90520-7](http://dx.doi.org/10.1016/0375-9601(86)90520-7)
- [3] Aspect A, Dalibard J and Roger G 1982 Experimental test of Bell's inequalities using time-varying analyzers *Phys. Rev. Lett.* **49** 1804–7 <http://dx.doi.org/10.1103/PhysRevLett.49.1804>
- [4] Gill R, Weihs G, Zeilinger A and Zukowski M 2002 No time loophole in Bell's theorem: the Hess-Philipp model is nonlocal *Proc. Natl. Acad. Sci. USA* **99** 14632–5 <http://dx.doi.org/10.1073/pnas.182536499>
- [5] Gill R, Weihs G, Zeilinger A and Zukowski M 2003 Comment on exclusion of time in the theorem of Bell *Europhys. Lett.* **61** 282–3 <http://dx.doi.org/10.1209/epl/i2003-00230-6>
- [6] Gill R D 2003 Time, finite statistics, and Bell's fifth position *Proceedings of Foundations of Probability and Physics-2* ed A Khrennikov (Växjö: Växjö University Press) pp 179–206
- [7] Larsson J-A and Gill R D 2004 Bell's inequality and the coincidence-time loophole *Europhys. Lett.* **67** 707–13 <http://dx.doi.org/10.1209/epl/i2004-10124-7>
- [8] Philipp W, Adenier G, Barraza-Lopez S and Hess K 2004 Local computer model emulating the results of the Pan *et al* experiment <http://www.arxiv.org/abs/quant-ph/0412209>
- [9] Calude C 1994 *Information and Randomness—An Algorithmic Perspective* (Berlin: Springer)
- [10] Svozil K 1990 The quantum coin toss—testing microphysical undecidability *Phys. Lett. A* **143** 433–7 [http://dx.doi.org/10.1016/0375-9601\(90\)90408-G](http://dx.doi.org/10.1016/0375-9601(90)90408-G)
- [11] Pan J-W, Bouwmeester D, Daniell M, Weinfurter H and Zeilinger A 2000 Experimental test of quantum nonlocality in three-photon Greenberger-Horne-Zeilinger entanglement *Nature* **403** 515–19
- [12] Einstein A, Podolsky B and Rosen N 1935 Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.* **47** 777–80 <http://dx.doi.org/10.1103/PhysRev.47.777>
- [13] Svozil K 2005 On counterfactuals and contextuality *AIP Conf. Proc.* **750**. (*Foundations of Probability and Physics-3*) ed A Khrennikov (Melville, NY: American Institute of Physics) pp 351–60 <http://dx.doi.org/10.1063/1.1874586>
- [14] Peres A 1978 Unperformed experiments have no results *Am. J. Phys.* **46** 745–7
- [15] Krenn G and Svozil K 1998 Stronger-than-quantum correlations *Found. Phys.* **28** 971–84 <http://dx.doi.org/10.1023/A:1018821314465>