Quantum correlations & beyond

http://tph.tuwien.ac.at/∼svozil/publ/2005-gdansk-pres.pdf

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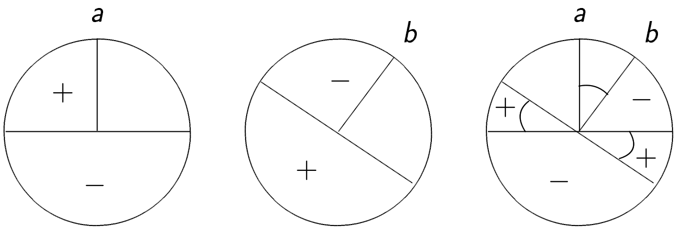
General setup

- Two measurement directions a and b of two dichotomic observables with values "-1" and "1" at two spatially separated locations.
- The measurement direction a at "Alice's location" is unknown to an observer "Bob" measuring b and *vice versa*.
- A two-particle correlation function $E(\theta)$ with $\theta = |a - b|$ is defined by averaging the product of the outcomes $O(a)_i, O(b)_i \in -1, 1$ in the ith experiment; i.e., $E(\theta) = (1/N) \sum_{i=1}^{N} O(a)_i O(b)_i$.

Classical correlations for two-particle "perfectly correlated" state

Assume uniform distribution of (opposite) "angular momentum" of the two particles; Alice measuring along angle a, Bob measuring along b:

$$E(a,b) = \frac{A_+(a,b) - A_-(a,b)}{2\pi} = \frac{2A_+(a,b) - 2\pi}{2\pi} = \\ = \frac{2}{\pi}|a-b| - 1 = \frac{2}{\pi}\theta - 1$$



Quantum correlations for two-particle singlet state

$$E(\theta) = 3/[j(j+1)]C(\theta)$$

with non-normalized

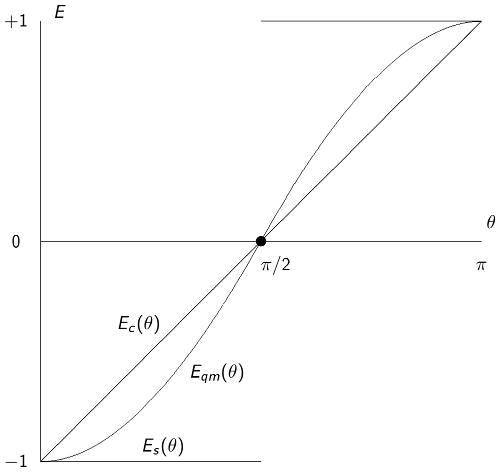
$$\begin{aligned}
C(\theta) &= \langle J=0, M=0 \mid \alpha \cdot \hat{J}^A \otimes \beta \cdot \hat{J}^B \mid J=0, M=0\rangle \\
&= \sum_{m,m'} \langle 00 \mid jm, j-m\rangle\langle jm', j-m' \mid 00\rangle \times \\
&\qquad \times {}^A\langle jm \mid^B \langle j-m \mid \alpha \cdot \hat{J}^A \otimes \beta \cdot \hat{J}^B \mid jm'\rangle^A \mid j-m'\rangle^B \\
&= \sum_{m,m'} \langle 00 \mid jm, j-m\rangle\langle jm', j-m' \mid 00\rangle \times \\
&\qquad \times \langle jm \mid \alpha \cdot \hat{J}^A \mid jm'\rangle\langle j-m \mid \beta \cdot \hat{J}^B \mid j-m'\rangle \\
&= \sum_{m,m'} \frac{(-1)^{j-m}(-1)^{j-m'}}{2j+1}\langle jm \mid \hat{J}_z^A \mid jm'\rangle\langle j-m \mid \beta \cdot \hat{J}^B \mid j-m'\rangle \\
&= \sum_{m,m'} \frac{(-1)^{j-m}(-1)^{j-m'}}{2j+1} m\delta_{mm'}\langle j-m \mid \beta \cdot \hat{J}^B \mid j-m'\rangle \\
&= \sum_{m} m\frac{(-1)^{2j-2m}}{2j+1}\langle j-m \mid \beta \cdot \hat{J}^B \mid j-m\rangle = \frac{1}{2j+1}\sum_m -m^2\beta_z = -\frac{1}{2j+1}\cos\theta\sum_{m=-j}^{j} m^2 \qquad \text{for } 0 \le \theta \le \pi \\
&= -\frac{j(j+1)}{3}\cos\theta \qquad \text{for } 0 \le \theta \le \pi \quad .
\end{aligned}$$

Stronger-than-quantum correlations

- S. Popescu and D. Rohrlich. Quantum nonlocality as an axiom. Foundations of Physics, 24(3):379-358, 1994. No violation of relativistic causality.
- G. Krenn and K.S., "Stronger-than-quantum correlations", Foundation of Physics 28(6), 971-984 (1998) [CrossRef DOI:10.1023/A:1018821314465]

	c	qm	s
$P^{=}(\theta) = 2P^{++}(\theta) = 2P^{--}(\theta)$	θ/π	$\sin^2(\theta/2)$	$H(2\theta/\pi - 1)$
$P^{\neq}(\theta) = 2P^{+-}(\theta) = 2P^{-+}(\theta)$	$1 - \theta/\pi$	$\cos^2(\theta/2)$	$H(1 - 2\theta/\pi)$
$E(\theta) = P^{=}(\theta) - P^{\neq}(\theta)$	$2\theta/\pi - 1$	$-\cos(\theta)$	$sgn(2\theta/\pi - 1)$

Stronger-than-quantum correlations cntd.



Stronger-than-quantum correlations cntd.

- More anti-coincidences of detector clicks between $0 < \theta < \pi/2$; more coincidences of detector clicks between $\pi/2 < \theta < \pi$; same-as-classical and quantum for $\theta = 0, \pi/2, \pi$.
- Clauser-Horne-Shimony-Holt (CHSH) inequality

$$|E(a,b) + E(a,b') + E(a',b) - E(a',b')| \leq 2$$

  for $a = \pi$,$a' = 3\pi/4$, $b = \pi/8$, $b' = 3\pi/8$ is violated by 4, the maximum value which is algebraically possible, and larger than the Tsirelson bound for quantum violations $2\sqrt{2}$.
- So far, only nonlocal model realizations — "Nature does not violate CHSH maximally." Why?
- not forbidden by relativistic causality, as long as there is parameter independence and mere outcome dependence.

Communication cost of breaking the Bell barrier (PRA, in press)

- Consider a single share λ, and an additional direction $\Delta(\delta)$, which is obtained by rotating $\hat{\lambda}$ clockwise around the origin by an angle δ which is a constant shift for all experiments. That is, $\Delta(\delta) = \lambda + \delta$.
- Alice's observable is given by

$$\alpha = \mathrm{sgn}(\hat{a} \cdot \hat{\lambda}) = \mathrm{sgn}\left[\cos(a - \lambda)\right].$$

- The bit communicated by Alice is given by

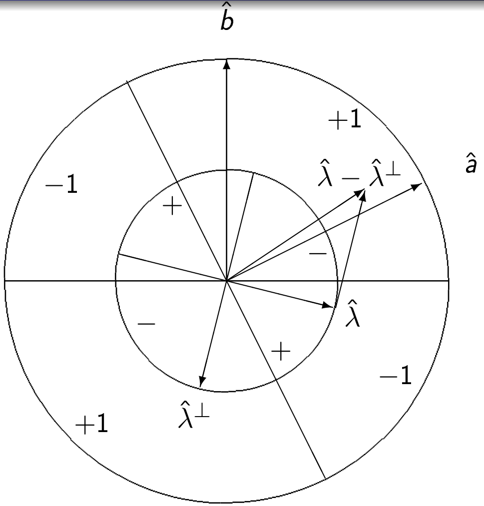
$$c(\delta) = \mathrm{sgn}(\hat{a}\cdot\hat{\lambda})\mathrm{sgn}\left[\hat{a} \cdot \hat{\Delta}(\delta)\right] = \mathrm{sgn}\left[\cos(a - \lambda)\right]\mathrm{sgn}\cos\left[a - \Delta(\delta)\right].$$

- Bob's observable is defined by

$$\beta(\delta) = \mathrm{sgn}[\hat{b} \cdot (\hat{\lambda} + c(\delta)\hat{\Delta}(\delta))].$$

(Motivated by B. F. Toner and D. Bacon, Physical Review Letters 91, 187904 (2003), URL http://dx.doi.org/10.1103/PhysRevLett.91.187904.)

Shift mechanism at work



Stronger-than-quantum correlations cntd.

- The associated correlation function is

$$E(\theta,\delta) = \begin{cases} -1 & \text{for} \quad 0 \le \theta \le \frac{\delta}{2}, \\ -1 + \frac{2}{\pi}(\theta - \frac{\delta}{2}) & \text{for} \quad \frac{\delta}{2} < \theta \le \frac{1}{2}(\pi - \delta), \\ -2(1 - \frac{2}{\pi}\theta) & \text{for} \quad \frac{1}{2}(\pi - \delta) < \theta \le \frac{1}{2}(\pi + \delta), \\ 1 + \frac{2}{\pi}(\theta - \pi + \frac{\delta}{2}) & \text{for} \quad \frac{1}{2}(\pi + \delta) < \theta \le \pi - \frac{\delta}{2}, \\ 1 & \text{for} \quad \pi - \frac{\delta}{2} < \theta \le \pi. \end{cases}$$

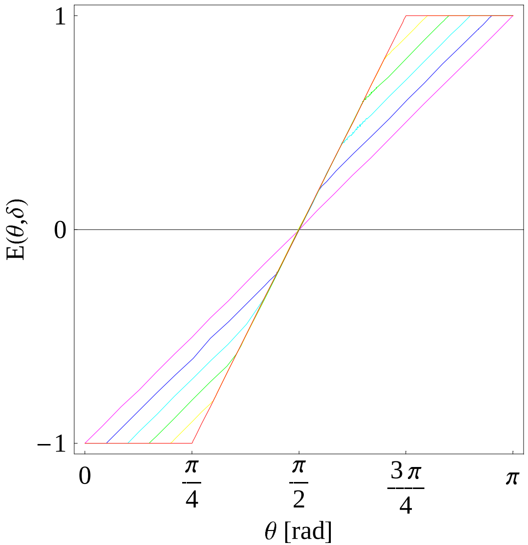
- Strongest correlation for $\delta = \pi/2$

$$E\left(\theta, \frac{\pi}{2}\right) = H\left(\theta - \frac{3\pi}{4}\right) - H\left(\frac{\pi}{4} - \theta\right) - 2\left(1 - \frac{2}{\pi}\theta\right) H\left(\theta - \frac{\pi}{4}\right) H\left(\frac{3\pi}{4} - \theta\right).$$

- Averaged correlation between $0 \le \delta \le \pi/2$

$$E(\theta) = \frac{2}{\pi}\int_0^{\pi/2} d\delta \; E(\theta,\delta) \frac{4}{\pi^2}\left[\left(\theta^2 - \frac{\pi^2}{4}\right) - 2H\left(\theta - \frac{\pi}{2}\right)\left(\theta - \frac{\pi}{2}\right)^2\right].$$

Stronger-than-quantum correlations cntd.



Stronger-than-quantum correlations cntd.

- For all nonzero δ, $E(\theta,\delta)$ correlates stronger than quantized systems for some values of θ.
- For $\delta=\pi/2$, the Clauser-Horne-Shimony-Holt inequality

$$|E(a,b)+E(a,b')+E(a',b)-E(a',b')|\leq 2$$

  for $a=\pi$,$a'=3\pi/4$, $b=0$, $b'=\pi/4$ is violated by 3, a larger value than the Tsirelson bound for quantum violations $2\sqrt{2}$.
- For $\delta=0$, the classical linear correlation function $E(\theta)=2\theta/\pi-1$ is recovered, as can be expected.
- Some open questions: Is it possible to obtain a maximal violation of CHSH with the exchange of a single bit? What about the exchange of a single (qu)bit between quanta?

Ansatz

Stefan Filipp and K.S., "Generalizing Tsirelson's Bound on Bell Inequalities Using a Min-Max Principle", PRL 93, 130407 (2004) [CrossRef DOI:10.1103/PhysRevLett.93.130407].

- Take an arbitrary Boole-Bell type inequality containing probabilities p_i and joint probabilities p_{ij}.
- Substitute

$$\begin{array}{lcl} p_1 & \rightarrow & q_1(\theta) = \frac{1}{2}\left[\mathbb{I}_2 + \sigma(\theta)\right] \otimes \mathbb{I}_2, \\ p_2 & \rightarrow & q_2(\theta) = \mathbb{I}_2 \otimes \frac{1}{2}\left[\mathbb{I}_2 + \sigma(\theta)\right], \\ p_{12} & \rightarrow & q_{12}(\theta,\theta') = \frac{1}{2}\left[\mathbb{I}_2 + \sigma(\theta)\right] \otimes \frac{1}{2}\left[\mathbb{I}_2 + \sigma(\theta')\right], \end{array}$$

  Note that, for hermitean operators A, B,
  $(A+B)^\dagger = A^\dagger + B^\dagger = A + B = (A+B)$
- Compute the Eigenvalues of the resulting hermitean operator and take the maximum eigenvalue
- The associated state violates the Boole-Bell type inequality maximally.

Qbounds for the Clauser-Horne inequality

- CH bounds on classical probabilities

$$-1 \leq p_{13} + p_{14} + p_{23} - p_{24} - p_1 - p_3 \leq 0$$

- Quantum CH operator

$$\begin{aligned}
&q_{13}(\alpha,\gamma) + q_{14}(\alpha,\delta) + q_{23}(\beta,\gamma) - q_{24}(\beta,\delta) - q_1(\alpha) - q_3(\gamma) \\
&= \tfrac{1}{2}\left[\mathbb{I}_2 + \sigma(\alpha)\right] \otimes \tfrac{1}{2}\left[\mathbb{I}_2 + \sigma(\gamma)\right] + \tfrac{1}{2}\left[\mathbb{I}_2 + \sigma(\alpha)\right] \otimes \tfrac{1}{2}\left[\mathbb{I}_2 + \sigma(\delta)\right] \\
&+ \tfrac{1}{2}\left[\mathbb{I}_2 + \sigma(\beta)\right] \otimes \tfrac{1}{2}\left[\mathbb{I}_2 + \sigma(\gamma)\right] - \tfrac{1}{2}\left[\mathbb{I}_2 + \sigma(\beta)\right] \otimes \tfrac{1}{2}\left[\mathbb{I}_2 + \sigma(\delta)\right] \\
&- \tfrac{1}{2}\left[\mathbb{I}_2 + \sigma(\alpha)\right] \otimes \mathbb{I}_2 - \mathbb{I}_2 \otimes \tfrac{1}{2}\left[\mathbb{I}_2 + \sigma(\gamma)\right],
\end{aligned}$$

- Eigenvalues

$$\lambda_{1,2,3,4}(\alpha,\beta,\gamma,\delta) = \frac{1}{2}\left(\pm\sqrt{1 \pm \sin(\alpha-\beta)\sin(\gamma-\delta)} - 1\right)$$

Qbounds for 2-partite 3-measurement configuration

- $I_{33} = p_{14}+p_{15}+p_{16}+p_{24}+p_{25}-p_{26}+p_{34}-p_{35}-p_1-2p_4-p_5 \leq 0.$
- Corresponding hermitean operator

$$\frac{1}{4}\begin{pmatrix} -4\sin^2\theta & 0 & 0 & 0 \\ 0 & -5-2\cos\theta-3\cos 2\theta+2\cos 3\theta & 4\cos^2\frac{\theta}{2} & 2\sin\theta+3\sin 2\theta-2\sin 3\theta \\ 0 & 4\cos^2\frac{\theta}{2} & -2(3+\cos 2\theta) & -2\sin\theta \\ 0 & 2\sin\theta+3\sin 2\theta-2\sin 3\theta & -2\sin\theta & 2\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}(4\cos\theta-3) \end{pmatrix}$$

- Maximum Q-violation is 4 at $\theta = \pi/3$

Thank you for your attention!