BOOK REVIEW: Quantum Logic in Algebraic Approach by Miklós Rédei

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Since the invention of quantum logic by Birkhoff & von Neumann there has been a steady interest in the topic. Quantum logic owes much of its current attention to the rise of experimental foundational physics. Many researchers interested in the foundation of physics begin to view this field as a way to understand the quantum better; or at least from a new, hitherto unknown viewpoint.

Yet much is still to be done to establish quantum logic as an indispensable tool, an area of research which not only offers new insights into established quantum physics but which progressively suggests totally novel phenomena out of its own body of formalism and knowledge. The physical community need to be convinced that the new algebraic formalisms and techniques which have to be learned are well worth the effort of digestion. Rédei's book *"Quantum Logic in Algebraic Approach"* [1] is an important contribution in this direction, as it represents a comprehensive introduction to the field.

The book starts out with concise expositions of the two main ingredients of quantum logic: the introductory chapters are devoted to quantum mechanics based on the Hilbert space formalism as well as to lattice theory. The spectral theorem as well as Gleason's theorem are reviewed as key theorems in connection with the physical concepts of observables and states.

In what follows, quantum logic is fully developed in the canonical manner as the lattice of of all closed linear subspaces of a Hilbert space, henceforth called Hilbert lattice. Already at this stage a very important theorem with respect to the "nonclassicality" of quantum mechanics is discussed: for Hilbert spaces of dimension greater than two no embedding exists which maps the associated Hilbert lattice into a "classical" Boolean algebra such that the logical (lattice) operations are preserved among comeasurable propositions. This fact, which is often referred to as the "Kochen-Specker" theorem, relates to the nonexistence of (certain properties of) two-valued measures which in term can be interpreted as the possibility to give values to elements of physical reality, irrespective of whether at all or in which context they have been measured (cf. below). Here a warning might be in order: Although there is little doubt that such an important result deserves a great share of attention, the novice reader might not be able to appreciate it at such an early state.

The book goes on with an outline of a semantic approach to physical theories, which is then applied to classical mechanics as well as to quantum mechanics. In this framework, "concrete quantum logic", is developed as the atomic, non-distributive, non-modular, orthomodular Hilbert lattice of projections on an infinite dimensional Hilbert space.

A theory of von Neumann algebras and von Neumann lattices is developed. This should enable the reader to comprehend and appreciate Birkhoff's and von Neumann's conception of quantum logic.

In the seventh chapter Rédei discusses, among other issues, a topic which seems to be not widely known in the history of quantum mechanics: von Neumann's loss of belief in the Hilbert space formalism of quantum mechanics. This is arguably one of the most important contributions to the present research, both historically and probabilistically, for it puts quantum logic into a new perspective. Some connections between possible connections between quantum logic and Cantorean set theory are also discussed.

After a chapter on the elementary theory of the quantum conditional connective, the hidden variable problem is reviewed. This problem is studied seriously and beyond the listing of historic events in an operator algebraic framework.

What neither Rédei nor the author of this review have foreseen is the possibility that a *dense subset* of the Hilbert lattice can be consistently assigned classical truth values independent of whether or not, or in which context, these "elements of physical reality" have been measured. This possibility, which has recently been argued by Mayer [2] and extended by Kent [3] and Clifton and Kent [4], makes conceivable non-contextual hidden parameter theories simulating essential operational aspects of quantum mechanics. Meyer's proof merely requires elementary combinatorics; it constructively demonstrates how to 'color' the rational unit sphere in threedimensional real Hilbert space such that for each orthogonal triad spanned by three vectors from the origin to the points of the sphere, one vector is 'colored' with 'yes' and the other two vectors are 'colored' with 'no'.

From a purely operational point of view, a "rationalized" quantum mechanics is indistinct from the usual real-valued formalism: given any nonzero measurement uncertainty ε and any 'non-colorable' Kochen-Specker graph $\Gamma(0)$, there exists another Graph (in fact, a denumerable infinity thereof) $\Gamma(\delta)$ which lies inside the range of measurement uncertainty $\delta \leq \varepsilon$ [and thus cannot be discriminated from the 'non-colorable' $\Gamma(0)$] which *can* be 'colored.' This is one of the rare cases where set theoretic assumptions—rationals versus reals—do make a difference: the first choice implies possible non-contextuality, the second choice context dependence. It remains to be seen whether these results can be extended to the Bell inequalities and the GHZ-theorem as well.

The next chapter of Rédei's book is devoted to the basic ingredients of the theory of local, algebraic relativistic quantum field theory. Bell-type correlations are introduced in operator algebraic terms. These considerations are developed further in a discussion of the problem of entanglement — the logical independence of two sub-quantum logics of a quantum logic. The last chapter is devoted to the analysis of whether or not superluminal correlations have common causes.

Rédei's book is very original and well written. It requires familiarity with functional analysis and Hilbert space quantum mechanics and thus might be suitable for mathematicians and theoretical physicists alike. But it also addresses issues of interest to philosophers of science with a background in the formalism, in particular with respect to the Birkhoff-von Neumann conception of quantum logic. To put it in Rédei's own words: *"The ideal reader I had in mind ... was a somewhat philosophically minded physicist with a strong respect and interest in mathematics."*

References

[1] Miklós Rédei. *Quantum Logic in Algebraic Approach*. Kluwer Academic Publishers, Dordrecht, Boston, London, 1998.

[2] David A. Meyer. Finite precision measurement nullifies the Kochen-Specker theorem. http://xxx.lanl.gov/abs/quant-ph/9905080, 1999.

[3] Adrian Kent. Non-contextual hidden variables and physical measurements. http://xxx.lanl.gov/abs/quant-ph/9906006, 1999.

[4] Rob Clifton and Adrian Kent. Simulating quantum mechanics by non-contextual hidden variables. http://xxx.lanl.gov/abs/quant-ph/9908031, 1999.