Foundations of Physics, Vol. 30, No. 7, 2000

Relativizing Relativity

K. Svozil[1]

Received February 4, 2000; revised May 8, 2000

Special relativity theory is generalized to two or more "maximal" signalling speeds. This framework is discussed in three contexts: (i) as a scenario for superluminal signalling and motion, (ii) as the possibility of two or more "light" cones due to the a "birefringent" vacuum, and (iii) as a further extension of conventionality beyond synchrony.

1. GENERAL FRAMEWORK

In what follows, we shall study *two* signal types with two different signal velocities generating two different sets of Lorentz frames associated with two types of "light" cones. (A generalization to an arbitrary number of signals is straightforward.) This may seem implausible and even misleading at first, since from two different "maximal" signal velocities only one can be truly maximal. Only the maximal one appears to be the natural candidate for the generation of Lorentz frames.

However, it may sometimes be physically reasonable to consider frames obtained by nonmaximal speed signalling. What could such *subluminal coordinates*, as they may be called, be good for?

(i) First of all, they may be useful for intermediate description levels[6, 54] of physical theory. These description levels may either be irreducible or derivable from some more fundamental level. Such considerations appear to be closely related to system science.

(ii) By analogy, we may also consider faster-than-light "signalling" generating *superluminal coordinates*.[45, 58] Presently, faster-than-light

[1] Institut für Theoretische Physik, Technische Universität Wien, Wiedner Hauptstraße 8-10/136, A-1040 Vienna, Austria. E-mail: svozil@tuwien.ac.at; www: http://tph.tuwien.ac.at/~svozil

0015-9018/00/0700-1001$18.00/0

"signalling" can, for instance, be realized by superluminal charge-current patterns; e.g., by the coordinated motion of aggregates of electrically charged particles.[13, 7] "Signals" of the above type cannot convey useful information and therefore cannot possibly be utilized to violate causality. But it could also be speculated that in the distant future signals of yet unknown type might be discovered which travel faster than light. In this view, the "second" type of "light" cone just has not been discovered yet; its discovery being independent of and beyond the scope of these considerations.

(iii) Thirdly, the standard debate of conventionality in relativity theory which concentrates on synchrony can be extended to arbitrary signalling speeds as well. This amounts to a splitting of relativity theory into a section dealing with operational meaningful conventions and another section expressing the physical content, in particular covariance; i.e., the form invariance of the equations of motion under the resulting space-time transformations.

In all these cases the following considerations may yield a clearer understanding of seemingly "paradoxical" effects such as time travel.[25, 46, 41] Thus it may not appear totally unreasonable to consider generalized system representations in which more than one signalling speeds are used to generate space and time scales. The transformation properties of such scales are then defined *relative to the signal* invoked.

2. EXTENDING CONVENTIONALITY

One of the greatest achievements of Einstein's theory of relativity is the operational approach to space and time: Already in Einstein's original article,[21] space as well as time scales are generated by physical procedures and observables which are based upon empirical phenomena and on intrinsically meaningful concepts.[15, 63, 59, 60, 51, 52, 61] Such a requirement is by no means trivial. For instance, different description levels may use different signals (e.g., sound, waves of any form, light,...).

Thereby, certain conventions have to be assumed, which again have an operational meaning by referring to purely physical terms. For instance, standard synchrony at spatially separated locations is conventionalized by "radar procedures;" i.e., by sending a signal back and forth between two spatially separated clocks. The conventionality of synchrony has been discussed, among others, by Reichenbach,[48] Grünbaum,[31, 30] Winnie,[66, 67] Malamet,[37] Redhead,[47] and Sarkar and Stachel[53] (cf. Ref. 34 for a review).

In what follows, we shall frequently use Einsteinian clocks based on "radar coordinates." Thereby, we shall first fix an arbitrary unit distance. Radar coordinate clocks use signals going back and forth between two reflective walls which are a predefined unit distance apart. Time is measured by the number of traversals of the signals between the walls. Thus, if different signals are used to define time scales, different time scales result.

In pursuing conventionalism further, it appears not unreasonable to *assume* the invariance of the speed of light as merely a convention rather than as an empirical finding. Historically, the special theory of relativity suggested a consistent manner to re-interpret spatial distances as time intervals. This made it necessary to have a conversion constant with the dimension of velocity. The most natural candidate was the speed of light, because it figured in the Maxwell equations, which are form invariant with respect to the Lorentz transformations based on the invariance of the speed of light. Consequently, the speed of light is no measurable quantity, but a fixed constant. Indeed, the International System of standard units(57) has implemented this approach.

The light cone structure already decides the transformation of space-time coordinates: Alexandrov's theorem(1–4, 49, 68, 33, 14, 29) (cf. below; see Refs. 12 and 36 for a review) states that the (affine) Lorentz transformations are a consequence of the invariance of the speed of light; a reasonable side assumption being the one-to-one mapping of coordinates.

Introducing relativistic space-time transformations as a consequence of conventions rather than of deep physics amounts to introducing relativity theory "upside down," since in retrospect and in standard reviews(21, 22, 55) the Michelson–Morley and Kennedy–Thorndike experiment is commonly presented as an *experimental finding* supporting the assumption of the invariance of light in all reference frames. Indeed, the very idea that the invariance of the two-way velocity of light is a mere convention might appear unacceptable. It seems rather uncommon and against well-tested principles to contend that the relativistic phenomena are mostly a matter of conventional representations. These phenomena are nowadays not only limited to the old Michelson Morley experiment, but include things like the velocity dependence of the life time of Kaons or other particles.

Yet, within a given level of description, an unavoidable self-referentiality should be acknowledged: all experiments are themselves based upon coordinates (e.g., clocks and scales) which operate with the very signals whose invariance is experienced. And since the physical world view is strongly based on agreed upon conventions of how to order observational data, can we introduce a new convention and obtain an equally consistent, but radically changed perspective of the world?

In contradistinction, the relativity principle, stating the *form invariance* of the physical laws under such Lorentz transformations, conveys the non-trivial physical content. In this way, special relativity theory is effectively split into a section dealing with geometric conventions and a different one dealing with the representation of physical phenomena.

Thereby, general covariance of the physical laws of motion can no longer be required globally. Indeed, form invariance will be satisfied only relative to a specific level of description; more precisely: relative to a particular class of space-time transformations.

To give a simple example: Maxwell's equations are not form invariant with respect to Lorentz-type transformations generated from the assumption of the invariance of the speed of sound; just as the description of onedimensional sound phenomena propagating with velocity $\bar{c}$ by $f(x-\bar{c}t) + g(x+\bar{c}t)$ is not invariant with respect to the usual Lorentz transformations. Yet, Maxwell's equations are form invariant with respect to the usual Lorentz transformations; just as $f(x-\bar{c}t)+g(x+\bar{c}t)$ is invariant with respect to Lorentz-type transformations generated from the assumption of the invariance of the speed of sound $\bar{c}$. [This can be checked by insertion into equation (4).]

With respect to a particular level of physical description, the time scale generated by the corresponding signal may be more appropriate than another if we adopt Poincaré's criterion[43] resembling Occam's razor: *"Time scales should be defined in such a way that the mechanical equations become as simple as possible. In other words, there is no way to measure time which is more correct than another one; the one commonly used is simply the most convenient."*

This is a radical departure from the requirement that the *fastest* signal should be used for coordinatization. Of course, today's fastest signal, light, is perfectly appropriate for today's fundamental description level of electromagnetism; the corresponding scales (generated by the assumption of invariance of the speed of light) leaving the Maxwell equations and other relativistic equations of motion form invariant. But that does by no means imply that different signals may not be more appropriate than light for different levels of physical description.

Indeed, if instead of light, sound waves or water waves would be assumed constant in all inertial frames, then very similar "relativistic effects" would result, but at a speed lower than the speed of light. This top-down approach to special relativity should be compared to still another bottom-up approach pursued, among others, by FitzGerald,[65] Jánossy,[35] Toffoli,[63] Erlichson,[23] Bell,[8, 10] Mansouri and Sexl,[38–40] Svozil,[59, 60] Shupe[56] and Günther.[32] There, relativistic forms are derived from "ether"-type theories.

We might also speculate that the vacuum might be a "birefringent" medium and that, for some unknown reason (e.g., the nonavailability of suitable detectors or the weakness of the signal), only one of the two vacuum "light" cones has been observed so far. As we shall mainly deal with the structural concepts of such findings, we shall not discuss these assumptions further.

3. TRANSFORMATION LAWS

In what follows we shall consider the transformation properties of coordinates between different reference frames; in particular between frames generated by two different "maximal" signalling speeds. To be more precise, let c denote the velocity of light. *Alexandrov's theorem*[1–4, 49, 68, 33, 14, 29] (cf. Refs. 12 and 36 for a review) states that one-to-one mappings φ: $\mathbf{R}^4 \to \mathbf{R}^4$ preserving the Lorentz–Minkowski distance *for light signals*

$$0 = c^2(t_x - t_y)^2 - (\mathbf{x} - \mathbf{y})^2 = c^2(t'_x - t'_y)^2 - (\mathbf{x}' - \mathbf{y}')^2$$

$x = (t_x, \mathbf{x})$, $y = (t_y, \mathbf{y}) \in \mathbf{R}^4$ are Lorentz transformations

$$x' = \varphi(x) = \alpha L x + a$$

up to an affine scale factor α. (A generalization to $\mathbf{R}^n$ is straightforward.) Hence, the Lorentz transformations appear to be essentially derivable from the invariance of the speed of light alone.

Consider now that we assume the convention that, for one and the same physical system and for reasons not specified, another arbitrary but different velocity $\bar{c}$ is invariant. As a result of Alexandrov's theorem, a different set of Lorentz-type transformation with c substituted by $\bar{c}$ is obtained. Of course, as can be expected, neither is $\bar{c}$ invariant in the usual Lorentz frames, nor is c invariant in the Lorentz-type frames containing $\bar{c}$: only the c-light cone appears invariant with respect to the transformations containing c; the $\bar{c}$-light cone is not. Conversely, only the $\bar{c}$-light cone appears invariant with respect to the transformations containing $\bar{c}$; the c-light cone is not.

Let us, for the sake of the argument, assume that $c < \bar{c}$. For all practical purposes, we shall consider two-way velocities (measured back and forth).

As argued before, we shall consider two sets of inertial frames Σ, $\bar{\Sigma}$ associated with c and $\bar{c}$, respectively. The set of all inertial frames Σ is constructed by *a priori* and *ad hoc* assuming that c is constant. The set of all inertial frames $\bar{\Sigma}$ is constructed by *a priori* and *ad hoc* assuming that $\bar{c}$ is constant.

The construction of Σ and $\bar{\Sigma}$ *via* Alexandrov's principle is quite standard. Since c and $\bar{c}$ are defined to be constant, two (affine) Lorentz transformations

$$x' = \varphi(x) = \alpha L x + a \qquad \text{and} \tag{1}$$

$$\bar{x}' = \bar{\varphi}(\bar{x}) = \bar{\alpha}\bar{L}\bar{x} + \bar{a} \tag{2}$$

result for Σ and $\bar{\Sigma}$, respectively. (In what follows, the affine factors α, $\bar{\alpha}$ are set to unity.) We shall also refer to these space and time scales as c-space, c-time, and $\bar{c}$-space, $\bar{c}$-time, respectively.

The rules for constructing space-time diagrams for the twodimensional problem (time and one space axis) are straightforward as well. The Lorentz transformations (1) and (2) for $a = \bar{a} = 0$ yield

$$\varphi_v(x) = (t', x_1', 0, 0) = \gamma\left(t - \frac{vx_1}{c^2}, x_1 - vt, 0, 0\right), \qquad \text{and} \tag{3}$$

$$\bar{\varphi}_{\bar{v}}(\bar{x}) = (\bar{t}', \bar{x}_1', 0, 0) = \bar{\gamma}\left(\bar{t} - \frac{\bar{v}\bar{x}_1}{\bar{c}^2}, \bar{x}_1 - \bar{v}\bar{t}, 0, 0\right) \tag{4}$$

with

$$\gamma = +\left(1 - \frac{v^2}{c^2}\right)^{-1/2} \qquad \text{and} \qquad \bar{\gamma} = +\left(1 - \frac{\bar{v}^2}{\bar{c}^2}\right)^{-1/2}$$

From now on, we shall write x and $\bar{x}$ for x_1 and $\bar{x}_1$, respectively. The second and third spatial coordinate will be omitted.

Consider faster-than-c velocities v in the range

$$c < v \leqslant \bar{c}$$

For this velocity range, the Lorentz transformations (3), in particular γ, become imaginary in the Σ-frames. Therefore, Σ cannot account for such velocities. For $\bar{\Sigma}$, these velocities are perfectly meaningful, being smaller than or equal to $\bar{c}$.

The x'- and t'-axis is obtained by setting $t' = 0$ and $x' = 0$, respectively. One obtains

$$t = \frac{vx}{c^2} \qquad \text{and} \qquad \bar{t} = \frac{\bar{v}\bar{x}}{\bar{c}^2} \tag{5}$$

for the x'- and $\bar{x}'$-axis, as well as

$$t = \frac{x}{v} \qquad \text{and} \qquad \bar{t} = \frac{\bar{x}}{\bar{v}} \tag{6}$$

for the t'- and $\bar{t}'$-axis, respectively.

In general, $c^2t^2 - x^2 \neq \bar{c}^2\bar{t}^2 - \bar{x}^2$, except for $c = \bar{c}$, and the coordinate frames cannot be directly compared. Thus the standard way of identifying unities does not work any longer.

We might, nevertheless, generalize relativity theory by *requiring* $c^2t^2 - x^2 = \bar{c}^2\bar{t}^2 - \bar{x}^2$. In this case, the identifications for unity are straightforward. In the following, a different approach is pursued.

One possibility is to proceed by constructing radar coordinates in the following operational way. Let us require that all frames Σ and $\bar{\Sigma}$ have the same origin. That is,

$$(t, x) = (0, 0) \Leftrightarrow (\bar{t}, \bar{x}) = (0, 0) \tag{7}$$

Furthermore, let us consider the intrinsic coordinatization of two coordinate frames $\sigma \in \Sigma$ and $\bar{\sigma} \in \bar{\Sigma}$ which are at rest with respect to each other. As a consequence of the standard Einstein synchronization conventions, two events which occur at the same c-time in σ also occur at the same $\bar{c}$-time in $\bar{\sigma}$. Note that this concurrence of synchrony is true only for the particular frames σ and $\bar{\sigma}$ and cannot be expected for all co-moving frames of Σ and $\bar{\Sigma}$. At this point, the preference of two frames σ, $\bar{\sigma}$ over others is purely conventional and does not reflect any “deep physics.”

Let us first assume that we proceed by fixing one and the same unit of distance for both coordinate systems; i.e., $x = \bar{x}$. In such a case, the radar time coordinate $\bar{t}$ can be expressed in terms of the radar time coordinate t by $\bar{t} = (\bar{c}/c)\, t$. This is illustrated in Fig. 1. In c-time 1, the faster signal with velocity $\bar{c}$ has been relayed back and forth the reflecting walls by a factor $\bar{c}/c$. Thus in summary, the transformation laws between σ and $\bar{\sigma}$ are

$$(\bar{t}(t, x), \bar{x}(t, x)) = \left(\frac{\bar{c}}{c}\, t, x\right) \tag{8}$$

A more general conversion between Σ and $\bar{\Sigma}$ involving moving coordinates is obtained by applying successively the inverse Lorentz transformation (3) and the Lorentz transformation (4) with velocities v and $\bar{w}$, respectively; i.e.,

$$(\bar{t}'', \bar{x}'') = \bar{\varphi}_{\bar{w}}(\bar{t}', \bar{x}') = \bar{\varphi}_{\bar{w}}\left(\frac{\bar{c}}{c}\, t', x'\right) \tag{9}$$

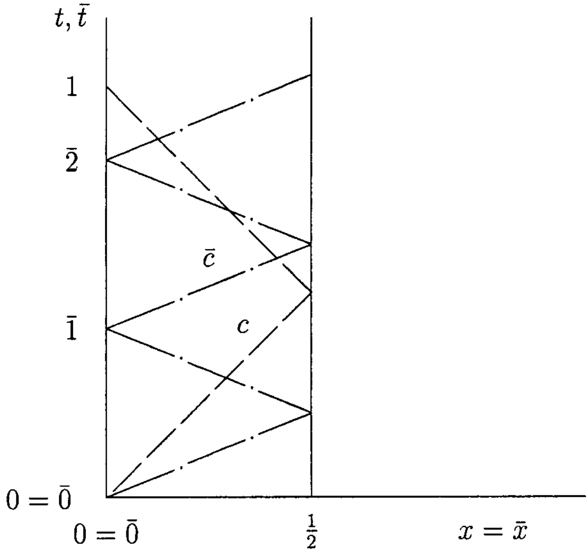

Fig. 1. Construction of radar time coordinates. The vertical lines represent mirrors for both signals at velocities c (denoted by a dashed line "– – –"), and $\bar{c}$ (denoted by "– · – · –").

with

$$(t', x') = \varphi_v^{-1}(t, x) = \gamma \left(t + \frac{vx}{c^2}, x + vt \right) \tag{10}$$

More explicitly,

$$\begin{aligned}
&(\bar{t}''(t, x), \bar{x}''(t, x)) \\
&= \gamma\bar{\gamma}\left[\frac{\bar{c}}{c}\left(t + \frac{vx}{c^2}\right) - \frac{\bar{w}}{\bar{c}^2}(x + vt), x + vt - \frac{\bar{w}\bar{c}}{c}\left(t + \frac{vx}{c^2}\right)\right] \\
&= \gamma\bar{\gamma}\left[t\left(\frac{\bar{c}}{c} - \frac{v\bar{w}}{\bar{c}^2}\right) + x\left(\frac{\bar{c}v}{c^3} - \frac{\bar{w}}{\bar{c}^2}\right), t\left(v - \frac{\bar{c}\bar{w}}{c}\right) + x\left(1 - \frac{\bar{c}v\bar{w}}{c^3}\right)\right]
\end{aligned} \tag{11}$$

Here, $v < c$ and $\bar{w} < \bar{c}$. As can be expected, for $c = \bar{c}$ and $v = \bar{w}$, Eq. (11) reduces to $(\bar{t}''(t, x), \bar{x}''(t, x)) = (t, x)$.

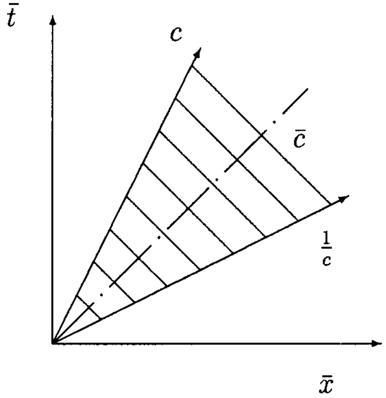

Fig. 2. Inertial frames of $\bar{\Sigma}$. The shaded area is forbidden for frames of Σ $(c < \bar{c})$.

Instead of identical space coordinates for two frames at rest with respect to each other, we could have chosen invariant time coordinates in both frames. A dual construction yields the transformation laws

$$(\bar{t}(t, x), \bar{x}(t, x)) = \left(t, \frac{c}{\bar{c}}\, x\right) \tag{12}$$

Let us now consider space-time diagrams for Σ and $\bar{\Sigma}$. Figure 2 depicts twodimensional coordinate frames generated for $\bar{\Sigma}$. The shaded region with the slope within $[c, 1/c]$ is not allowed for Σ. It corresponds to faster-than-c frames.

Figure 3 draws a representation of the sets of frames Σ and $\bar{\Sigma}$ in the set of all affine frames, denoted by a square.

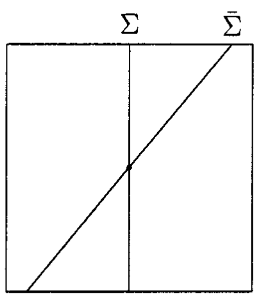

Fig. 3. The set of all inertial frames of Σ and $\bar{\Sigma}$ in the set of all affine frames. The intersection between Σ and $\bar{\Sigma}$ represents frames of equal synchrony.

4. QUASI TIME PARADOXES AND THEIR RESOLUTION

Since $\bar{c} > c$, superluminal signalling by any velocity v with $\bar{c} \geqslant v > c$ with respect to c is an option for Σ. This could, at least from a straightforward point of view, result in quasi-time paradoxes, such as Tolman's[64, 11, 45, 46] or Gödel's paradoxes.[25–27] They originate from the fact that, given superluminal signalling, signalling back in c-time is conceivable, making a diagonalization argument[17, 18, 50, 42] similar to the classical liar[5] possible.[62] Stated pointedly: given free will, this would enable an agent to send a signal backwards in time if and only if the agent has not received this message before (or, in a more violent version, kill the agent's own grandfather in early childhood).[41] Likewise, this would allow an agent to become very knowledgeable, powerful and rich, which is not necessarily paradoxical.

To illustrate the quasi-paradoxical nature of the argument, let us consider a concrete example. Assume as the two signalling speeds c and $\bar{c}$ the speed of sound and the speed of light, respectively. Let us further assume that there exist intelligent beings—let us call them "soundlanders"—capable of developing physics in their "ether"-medium.[56, 59, 60, 20, 32] For them, sound would appear as a perfectly appropriate phenomenon to base their coordinate frames upon. To give an example, the wave equation

$$\Delta u = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$$

governing the propagation of sound of velocity c is invariant with respect to (3). For this reason, to soundlanders the speed of sound might appear as the "natural" speed to base their system of units upon.

What if the soundlanders discover supersonic shock waves propagating through their elastic medium, or sonoluminiscence; i.e., the creation of signals at supersonar speed $\bar{c}$? Surely, because of the conceivable paradoxes discussed before, this would result in a denial of the experimental findings at first and in a crisis of (theoretical sound) physics later. Figure 4 depicts the construction of a quasi-time paradox, as perceived from the inertial frame Σ generated by sound and the inertial frame $\bar{\Sigma}$ generated by light.

However, as can be expected, when viewed from $\bar{\Sigma}$, the seemingly "paradoxical" process perceived by Σ is not paradoxical at all. It appears that one resolution of the paradoxes is to switch the level of observation and take the perspective that the "true physics" is not based upon sound but on electromagnetic phenomena. After all, sound waves result from the coordinated motion of aggregates of atoms or molecules, which in turn is dominated by the electromagnetic forces. In this extrinsic view, the "sound physics" of the "soundlanders" is a representation of the phenomena at

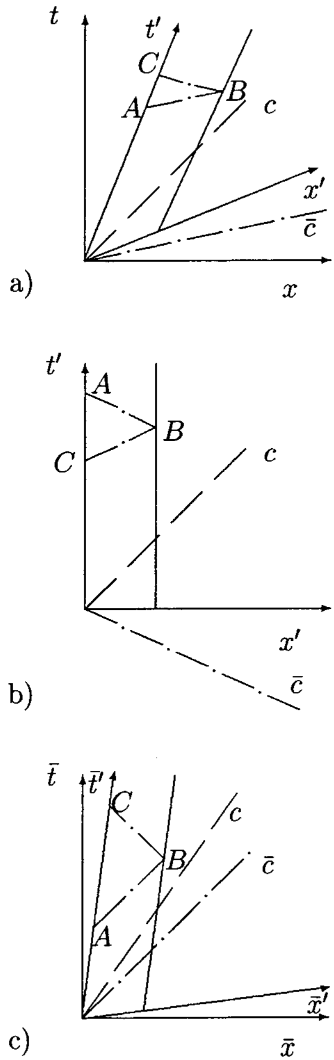

Fig. 4. (a) Quasi-time paradox as perceived from Σ-frame (t, x). There is no apparent paradox here, because since $t_A < t_B < t_C$, no information flows backward in time. (b) Quasi-time paradox reveals itself when perceived from Σ-frame (t', x'). Information appears to flow backward in time, since $t_A > t_B > t_C$. (c) Resolution of the time paradox in $\bar{\Sigma}$-frame $(\bar{t}, \bar{x})$. In all $\bar{\Sigma}$-frames, $\bar{t}_A < \bar{t}_B < \bar{t}_C$.

an intermediate level of description.(6, 54) Since from that viewpoint, the appropriate signalling speed is electromagnetic radiation at velocity $\bar{c}$, paradoxes disappear. Thus any attempt to construct paradoxes at the intermediate level of sound signals is doomed to fail because that level of description turns out to be inappropriate for the particular purpose.

This extrinsic viewpoint is juxtaposed by the intrinsic viewpoint(15, 51, 52, 59–61, 63) of the "soundlanders" pretending to maintain their intermediate level of "sound physics." For them, paradoxes are not realizable because certain procedures or actions are not operational. This amounts to the resolution of time paradoxes by the principle of self-consistency[24] as already discussed, for instance, in Nahin's monograph [Ref. 41, p. 272].

In analogy to the above comparison between sound and light signals, one could consider superluminal signalling by some (admittedly highly speculative) source of yet unknown origin. Very likely, any such signalling would operate with a hitherto unknown type of interaction, since any signal might be associated with a disturbance a sender causes at the receiver by an interaction. As a consequence, conventionality suggests that this new signal is the "natural candidate" for defining space time scales and the Lorentz-type transformations between different frames. It may not be totally unreasonable to speculate that the laws governing this new interactions are form invariant under the new, superluminal Lorentz transformation containing the faster signalling speed instead of the speed of light. In such a scenario, many of the known fundamental laws, such as the electroweak and strong interactions, may then appear as effectively (ab initio) derivable from the new type of interaction. (Just as sound phenomena have an electromagnetic origin.)

The new Lorentz transformations may then serve the purpose of re-establishing a *unique* mode of conceptually connecting different space time points. After all, one may justifiably argue that a world with multiple maximal speeds and multiple relativistic descriptions could merely exist if the different speeds referred to fields entirely unrelated to each other. In such a world, no mixed phenomena would exist. But if the phenomena are interrelated or one is effectively derivable from the other, the symmetric picture of an arbitrariness of the choice of "natural" units and speeds breaks down.

5. CONCLUDING REMARKS

As speculative as the above considerations may appear, they can be brought forward consistently. Even if exotic scenarios such as a birefringent vacuum appears highly unlikely, some lessons for the presentation and interpretation of standard relativity theory, in particular the splitting of conventions from the form invariance of the physical laws, can be learned.

Theory—in the author's opinion for the worse—tends to exert a conservative influence in declining that faster-than-light or "superluminal" information communication and travel of the type *"breakfast on Earth, lunch on Alpha Centauri, and home for dinner with your wife and children,* *not your great-great-great grandchildren*"[44] is conceivable. Accordingly, any experimental, empirical claim of allegedly superluminal phenomena is confronted with the strongest resistance from the theoretical orthodoxy, pretending the principal impossibility for superluminal communication.

The author is not convinced that, as of today, there is reason to believe that there is experimental evidence of faster-than-light communication via tunnelling or other phenomena. Yet, one cannot know when, if ever, superluminal phenomena may be discovered. (In the author's opinion these would most probably show up in an allegedly nonpreservation of energy and momentum; very much in the same way as sonoluminiscence may be viewed from the description level of sound.) Hence, one purpose of this study has been the attempt to free experiment from the pressure of the theoretical orthodoxy. "Superluminal" signalling *per se* is accountable for and does not necessarily imply "phenomenologic" inconsistency.

From a system theoretic point of view, a generalized principle of overall consistency of the phenomena might be used to demonstrate that too powerful agents would become inconsistent. As a consequence, the predictive power as well as the physical operationalizability (command over the phenomena) is limited by this consistency requirement. Events which may appear undecidable and uncontrollable to an intrinsic observer bound by incomplete knowledge may be perfectly controllable and decidable with respect to a more complete theory. In such a framework, different signalling speeds, in particular also superluminal signalling, can well be accommodated within a generalized theory of relativity. They do not at all mean inconsistencies but just refer to different levels of physical descriptions and conventions which have to be carefully accounted for.

ACKNOWLEDGMENTS

The author is grateful for discussions with and suggestions and comments by John Casti, Georg Franck, Günther Krenn, Volkmar Putz, Otto Rössler, Johann Summhammer, and Helmuth Urbantke. However, almost needless to say, their opinions do not necessarily coincide with the approach expressed in this article; nor should they be blamed for any misconception and fallacy of the author.

REFERENCES

1. A. D. Alexandrov, "On Lorentz transformations," *Uspehi Mat. Nauk*. **5**(3), 187 (1950).
2. A. D. Alexandrov, "A contribution to chronogeometry," *Canadian J. Math*. **19**, 1119–1128 (1967).

3. A. D. Alexandrov, “Mappings of spaces with families of cones and space-time transformations,” *Annali die Matematica Pura ed Applicata* **103**, 229–257 (1967).
4. A. D. Alexandrov, “On the principles of relativity theory,” in *Classics of Soviet Mathematics*, Vol. 4, A. D. Alexandrov, Selected Works, pp. 289–318 (1996).
5. A. R. Anderson, “St. Paul’s epistle to Titus,” in *The Paradox of the Liar*, R. L. Martin, ed. (Yale University Press, New Haven, 1970). The Bible contains a passage which refers to Epimenides, a Crete living in the capital city of Cnossus: “One of themselves, a prophet of their own, said, ‘Cretans are always liars, evil beasts, lazy gluttons.’ ”—St. Paul, Epistle to Titus I (12–13).
6. Philip W. Anderson, “More is different,” *Science* **177**(4047), 393–396 (August 1972).
7. Houshang Ardavan, “A speed-of-light barrier in classical electrodynamics,” *Phys. Rev. D* **29**(2), 207–215 (1984).
8. John S. Bell, “How to teach special relativity,” *Progr. Sci. Culture* **1**(2) (1976). Reprinted in Ref. 9, pp. 67–80.
9. John S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987).
10. John S. Bell, “George Francis FitzGerald,” *Physics World* **5**(9), 31–35 (September 1992), abridged version by Denis Weaire.
11. G. A. Benford, D. L. Book, and W. A. Newcomb, “The tachyonic antitelephone,” *Phys. Rev. D* **2**(2), 263–265 (1979).
12. Walter Benz, *Geometrische Transformationen* (BI Wissenschaftsverlag, Mannheim, 1992).
13. B. M. Bolotovskii and V. L. Ginzburg, “The Vavilov–Cerenkov effect and the Doppler effect in the motion of sources with superluminal velocity in vacuum,” *Sov. Phys. Uspekhi* **15**(2), 184–192 (1972). [*Usp. Fiz. Nauk* **106**, 577 (1972)].
14. H. J. Borchers and G. C. Hegerfeldt, “The structure of space-time transformations,” *Comm. Math. Phys.* **28**, 259–266 (1972).
15. R. J. Boskovich, “De spacio et tempore, ut a nobis cognoscuntur” (Vienna, 1755); English translation in Ref. 16.
16. R. J. Boskovich, “De spacio et tempore, ut a nobis cognoscuntur,” in *A Theory of Natural Philosophy*, J. M. Child, ed. (Open Court, 1922; MIT Press, Cambridge, MA, 1966), pp. 203–205.
17. Georg Cantor, “Beiträge zur Begründung der transfiniten Mengenlehre. I,” *Math. Annalen* **46**, 481–512 (1895). Reprinted in Ref. 19.
18. Georg Cantor, “Beiträge zur Begründung der transfiniten Mengenlehre. II,” *Math. Annalen* **49**, 207–246 (1897).
19. Georg Cantor, *Gesammelte Abhandlungen* (Springer, Berlin, 1932).
20. John L. Casti, private communication, May 1996.
21. Albert Einstein, “Zur Elektrodynamik bewegter Körper,” *Ann. Physik* **17**, 891–921 (1905).
22. Albert Einstein, *Grundzüge der Relativitätstheorie*, 1st edn. (Vieweg, Braunschweig, 1956).
23. Herman Erlichson, “The rod contraction-clock retardation ether theory and the special theory of relativity,” *Am. J. Phys.* **41**, 1068–1077 (1973).
24. John Friedman, Michael S. Morris, Igor D. Novikov, Fernando Echeverria, Gunnar Klinkhammer, Kip S. Thorne, and Ulvi Yurtsever, “Cauchy problem in spacetimes with closed timelike curves,” *Phys. Rev. D* **42**(6), 1915–1930 (1990).
25. Kurt Gödel, “An example of a new type of cosmological solutions of Einstein’s field equations of gravitation,” *Rev. Mod. Phys.* **21**, 447–450 (1949). Reprinted in Ref. 28, pp. 190–198.
26. Kurt Gödel, “A remark about the relationship between relativity theory and idealistic philosophy,” in *Philosopher-Scientist*, *Albert Einstein* (Tudor, New York, 1949; reprinted in Ref. 28, pp. 202–207), pp. 555–561.

27. Kurt Gödel, in *Collected Works. Unpublished Essays and Lectures*, S. Feferman, J. W. Dawson, Jr., W. Goldfarb, C. Parsons, and R. M. Solovay, eds. (Oxford University Press, Oxford, 1990), Vol. III.
28. Kurt Gödel, in *Collected Works. Publications 1938–1974*, S. Feferman, J. W. Dawson, Jr., S. C. Kleene, G. H. Moore, R. M. Solovay, and J. van Heijenoort, eds. (Oxford University Press, Oxford, 1990), Vol. II.
29. Robert Goldblatt, *Orthogonality and Spacetime Geometry* (Springer, New York, 1987).
30. A. Grünbaum. *Philosophical Problems of Space of Time* (Boston Studies in the Philosophy of Science, Vol. 12), 2nd ed. (Reidel, Dordrecht, 1974).
31. A. Grünbaum, W. Salmon, B. van Fraassen, and A. Janis, "A panel discussion of simultaneity by slow clock transport in the special and general theories of relativity," *Phil. Sci.* **36**, 1–81 (1969).
32. Helmut Günther, *Grenzgeschwindigkeiten und ihre Paradoxa* (Teubner, Stuttgart, Leipzig, 1996).
33. G. C. Hegerfeldt, "The Lorentz transformations: Derivation of linearity and scale factor," *Nuovo Cimento A* **10**, 257–267 (1972).
34. Allen I. Janis, "Conventionality of simultaneity," in *Stanford Encyclopedia of Philosophy. Winter 1998 Edition* (The Metaphysics Research Lab Center for the Study of Language and Information. Ventura Hall Stanford University Stanford, CA 94305-4115, 1998. Internet address: http://plato.stanford.edu/entries/spacetime-convensimul/).
35. L. Jánossy, *Theory of Relativity Based on Physical Reality* (Akadémiai Kiadó, Budapest, 1971).
36. June A. Lester, "Distance preserving transformations," in *Handbook of Incidence Geometry*, Francis Buekenhout, ed. (Elsevier, Amsterdam, 1995).
37. D. Malament, "Causal theories of time and the conventionality of simultaniety," *Noûs* **11**, 293–300 (1977).
38. Reza Mansouri and Roman U. Sexl, "A test of special relativity: I. Simultaneity and clock synchronization," *Gen. Relativ. Gravit.* **8**(7), 497–513 (1977).
39. Reza Mansouri and Roman U. Sexl, "A test of special relativity: II. First order tests," *Gen. Relativ. Gravit.* **8**(7), 513–524 (1977).
40. Reza Mansouri and Roman U. Sexl, "A test of special relativity: III. Second-order tests," *Gen. Relativ. Gravit.* **8**, 809–814 (1977).
41. Paul J. Nahin, *Time Travel*, 2nd ed. (AIP Press and Springer, New York, 1998).
42. Piergiorgio Odifreddi, *Classical Recursion Theory* (North-Holland, Amsterdam, 1989).
43. Henri Poincaré, *La valeur de la science* (Flammarion, Paris, 1905); English translation: *The Value of Science*, 1907; German translation: *Der Wert der Wissenschaft*, 1906.
44. H. E. Puthoff, "Can the vacuum be engineered for spaceflight applications?" paper presented at the NASA Breakthrough Propulsion Physics Conference at Lewis Research Center, 1997.
45. Erasmo Recami, "Classical tachyons and possible applications," *Nuovo Cimento* **9**(6), 1–178 (1986).
46. Erasmo Recami, "Tachyon kinematics and causality: A systematic thorough analysis of the tachyon causal paradoxes," *Found. Phys.* **17**(3), 239–296 (1987).
47. Michael Redhead, "The conventionality of simultaneity," in *Philosophical Problems of the Internal and External Worlds*, J. Earman, A. Janis, G. Massey, and N. Rescher, eds. (University of Pittsburgh Press and Universitätsverlag Konstanz, Pittsburgh and Konstanz, 1993), pp. 103–128.
48. H. Reichenbach, *The Philosophy of Space & Time* (Dover, New York, 1958).
49. Alfred A. Robb, *A Theory of Time and Space* (Cambridge University Press, Cambridge, 1914).

50. Hartley Rogers, Jr., *Theory of Recursive Functions and Effective Computability* (MacGraw–Hill, New York, 1967).
51. Otto E. Rössler, "Endophysics," in *Real Brains*, *Artificial Minds*, John L. Casti and A. Karlquist, eds. (North-Holland, New York, 1987), p. 25.
52. Otto E. Rössler, *Endophysics. Die Welt des inneren Beobachters* (with a foreword by Peter Weibel) (Merwe Verlag, Berlin, 1992).
53. Sahotra Sarkar and John Stachel, "Did Malament prove the non-conventionality of simultaneity in the special theory of relativity?" *Phil. Sci.* **66**, 208–220 (1999).
54. Silvan Schweber, "Physics, community, and the crisis in physical theory," *Phys. Today*, pp. 34–40, November 1993.
55. R. U. Sexl and H. K. Urbantke, *Relativität*, *Gruppen*, *Teilchen* (Springer, Vienna, 1976).
56. Michael A. Shupe, "The Lorentz-invariant vacuum medium," *Am. J. Phys.* **53**(2), 122–132 (1985).
57. The International System (SI) of units assumes light to be constant. It was decided in 1983 by the General Conference on Weights and Measures that the accepted value for the speed of light would be exactly 299,792,458 meters per second. (The meter thus is now *defined* as the distance traveled by light in a vacuum in 1/299,792,458 second, independent of the inertial system.)
58. Roderik I. Sutherland and John R. Shepanski, "Superluminal reference frames and generalized Lorentz transformations," *Phys. Rev. D* **33**(8) (1986).
59. Karl Svozil, "Connections between deviations from Lorentz transformation and relativistic energy-momentum relation," *Europhys. Lett.* **2**, 83–85 (1986).
60. Karl Svozil, "Operational perception of space-time coordinates in a quantum medium," *Nuovo Cimento B* **96**, 127–139 (1986).
61. Karl Svozil, *Randomness & Undecidability in Physics* (World Scientific, Singapore, 1993).
62. Karl Svozil, "Consistent use of paradoxes in deriving contraints on the dynamics of physical systems and of no-go-theorems," *Found. Phys. Lett.* **8**(6), 523–535 (1995).
63. T. Toffoli, "The role of the observer in uniform systems," in *Applied General Systems Research*, G. Klir, ed. (Plenum, New York, 1978).
64. R. C. Tolman, *The Theory of Relativity of Motion* (University of California Press, Berkeley, 1917).
65. E. Whittaker, "George Francis Fitzgerald," *Sci. Am.* **185**(5), 93–98 (1953).
66. J. Winnie, "Special relativity without one-way velocity assumptions: Part I," *Phil. Sci.* **37**, 81–99 (1970).
67. J. Winnie, "Special relativity without one-way velocity assumptions: Part II," *Phil. Sci.* **37**, 223–238 (1970).
68. E. C. Zeeman, "Causality implies the Lorentz group," *J. Math. Phys.* **5**, 490–493 (1964).