

## Time paradoxes reviewed

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### Abstract

Classical time paradoxes can be used to argue against controllable faster-than-light signalling. Alternatively, they can be resolved by the “switching principle”, but then the lapse of time for an observer embedded in a reference frame moving faster than light as seen by a subluminal observer is not invariant with respect to (orthochronous) Lorentz transformations.

Since so far no physical objects conveying classical information faster-than-light have been found, no attempt is undertaken to promote “tachyons” here. Yet, objects conveying classical information faster-than-light or even observers moving superluminally can be used for a *reductio ad absurdum*; i.e., for the construction of paradoxes, whose consistent resolution results in physical no-go theorems.

Assume two observers A and B which can communicate both faster-than-light as well as *backward* in time; i.e., a faster-than-light signal traverses the distance  $x_{AB}$  between them in time  $t_{AB}$  such that  $x_{AB}^2/t_{AB}^2 > 1$  as well as  $t_{AB} < 0$  (the velocity of light  $c = 1$ )<sup>2</sup>. Then, the observer A might emit a signal at time  $t_A$ , which arrives at the observer B in  $t_B$ , where it is reflected and is back at the observer A at a time  $t_{A'} < t_A$ , i.e., *before* observer A has emitted the original signal (Fig. 1). If one performs a “diagonal-

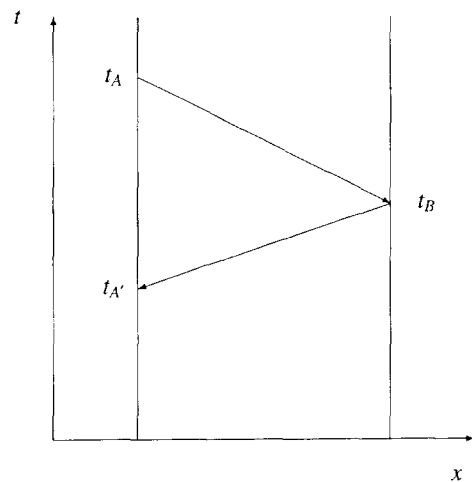


Fig. 1. Configuration allowing a classical time paradox.

ization”, i.e., if one assumes that observer A emits a signal at time  $t_A$  if and only if *no* signal is absorbed at  $t_{A'}$ ; observer A emits *no* signal at time  $t_A$  if and only if a signal is absorbed at  $t_{A'}$ , one ends up with the simplest form of time paradox [1–4].

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<sup>2</sup> It is always possible to transform a worldline of a faster-than-light signal with time component  $t_{AB} > 0$  into one with  $t_{AB} < 0$  by some (orthochronous) Lorentz transformation, i.e., by changing the coordinate system.

The syntactic structure of this paradox closely resembles Cantor's diagonalization method (based on the ancient liar paradox<sup>3</sup>, which has been applied by Gödel, Turing and others for undecidability proofs in a recursion theoretic setup [6,7]. The strategy in the recursion theoretic context is to avoid the appearance of a paradox by claiming (stronger: requiring) overall consistency, resulting in no-go theorems; i.e., in the postulate of the impossibility of any method, procedure or device which would have the potentiality to cause a paradox. Among the many impossible objects giving rise to paradoxes are such innocent devices as a "halting algorithm" computing whether or not another (arbitrary computable) algorithm produces a particular output; or an algorithm identifying (from the class of computable algorithms) another algorithm by input-output experiments. In physical terms, the appearance of time paradoxes would mean the impossibility of faster-than-light signalling in the discussed setup.

Yet, the impossibility of faster-than-light *signalling* does not necessarily mean the absence of any *entanglement* or *correlation* spreading faster-than-light. Faster-than-light effects may appear only after reconstruction of the event history, such that they can never give rise to contradictions. To preserve overall consistency it suffices to assume the *uncontrollability* of absorption, reflection and emission events in the above setup. For, if it were impossible there to purposefully stimulate the emission or the absorption of the signal travelling faster-than-light, then no paradox could be constructed. In other words, due to intrinsically uncontrollable and "random" events, the above diagonalization may not be physically operational. This feature might indeed be the basis of the conspicuous "peaceful coexistence" [8,9] between relativity theory and quantum mechanics, in particular with respect to non-local quantum correlations. A related argument seems to have been put forward by Professor Y. Aharonov at a talk delivered at the Einstein centennial [10]. Also Deutsch's resolution of time paradoxes [11] in the context of Everett's interpretation of quantum mechan-

ics seems to be of the same flavour when translated into the Copenhagen interpretation.

There is something to be learned from these considerations both for physics and the computer sciences. For physics, the message of recursion theory is that for a (sufficiently strong) computable system to be consistent means to be incomplete or undecidable. In phenomenological terms, not all events might be controllable by intrinsic [7,12–15] operations and devices.

Another way of perceiving time paradoxes from faster-than-light signalling (cf. Refs. [1,16,17,10]) is based on the standard quantum field theoretic reinterpretation of negative energy particles moving forward in time as positive energy particles moving backward in time, which in turn are reinterpreted [18,19] as positive energy *antiparticles* moving forward in time with reversed charges and velocities [20,21,3]. In this process, any emission is reinterpreted as absorption and vice versa. This "switching principle" guarantees that faster-than-light effects cannot even in principle give rise to time paradoxes such as the above one.

What the "switching principle" cannot guarantee, however, is the proper reinterpretation of the lapse of time for an observer moving faster-than-light. This can be demonstrated by considering an inertial frame  $\Sigma$  and an observer embedded therein co-moving faster-than-light. The assumption that *observers* move faster-than-light goes beyond superluminal signalling. Such "*anti*"observers could be thought of as built out of "tachyons." When properly reinterpreted in terms of the "switching principle" "*anti*"observers would have positive rest mass with respect to observers moving at subluminal speeds [20,3]. (Stated pointedly, the "switching principle" effectively prevents the observer to "become imaginary".)

Of course, no Lorentz transformation can produce an inertial frame  $\Sigma$  (and an observer therein) which moves faster-than-light from a subluminal inertial frame. Thus either a superluminal  $\Sigma$  has to be assumed a priori or an extended model of relativity (cf. Ref. [3]) has to be considered. In extended relativity, transformations from subluminal to superluminal inertial frames (and vice versa) are allowed. As has been stated already at the beginning, there is, of course, no evidence of any physical principle or observable effect which would promote such an extension of relativity theory.

Consider a second – subluminal – inertial frame  $I$

<sup>3</sup> The Bible contains a passage, which refers to Epimenides, a Crete living in the capital city of Cnossus: "One of themselves, a prophet of their own, said, 'Cretans are always liars, evil beasts, lazy gluttons.'" — St. Paul, Epistle to Titus 1 (12-13). For more details, see Ref. [5].

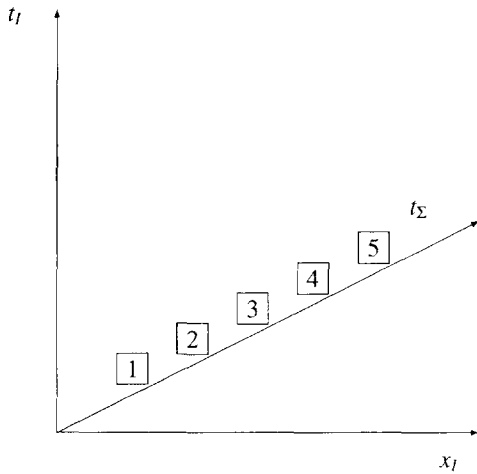


Fig. 2. Time display for an observer embedded in a frame of reference which moves faster than light (arbitrary units,  $c = 1$ ).

such that with respect to I, the world line of  $\Sigma$  “proceeds” forward in time. In other words, if the observer embedded in  $\Sigma$  constructs some clock (e.g., a light clock) to measure the lapse of time, this lapse of time will be in the same direction (it will not be reversed) when seen from I (Fig. 2).

Now apply to I an (orthochronous) Lorentz transformation to obtain a third inertial frame  $\bar{I}$ , for which the lapse of time in  $\Sigma$  is reversed. Due to the “switching principle” this reversed lapse of time is reinterpreted as a lapse forward in time, along with an antiparticle representation of  $\Sigma$ . But what happens to the lapse of time of the intrinsic “anti”observer in  $\Sigma$ ? In particular, what happens to the intrinsic time of the “anti”observer? The lapse of time of an intrinsic observer cannot be reversed just because another reference frame  $\bar{I}$  has been invoked. The observer’s time display shows the unequivocal evidence that the “switching procedure” has not affected the intrinsic time of the observer (Fig. 3). This observer-related paradox, it seems, can only be consistently avoided by either abandoning faster-than-light frames altogether, or by assuming that no objective lapse of time exists. This latter alternative, of course, has been the main motive behind Gödel’s solutions of the Einstein field equations exhibiting closed time-like trajectories [22,23], suggesting that, under certain conditions, in the theory of relativity, the concept of free will might lose any justification.

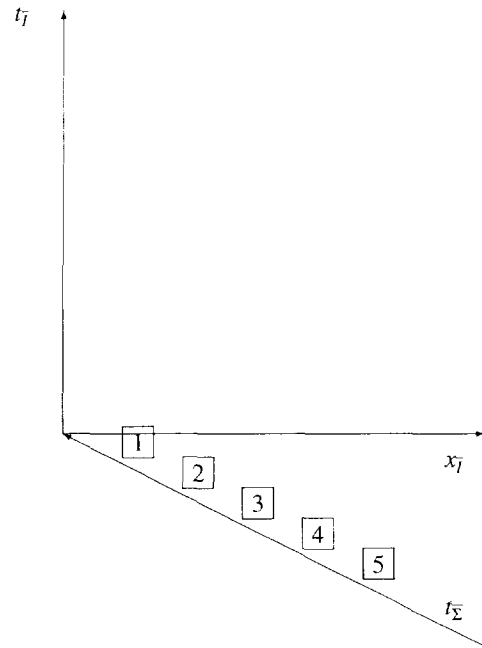


Fig. 3. The same configuration as before (Fig. 2), but perceived from another reference frame  $\bar{I}$  obtained from I by (orthochronous) Lorentz transformation and after reinterpretation by the “switching principle”.

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