# Time generated by intrinsic observers<sup>\*</sup>

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We shortly review the construction of knowledge by intrinsic observers. Intrinsic observers are embedded in a system and are inseparable parts thereof. The intrinsic viewpoint has to be contrasted with an extrinsic, "God's eye" viewpoint, from which the system can be observed externally without in any way changing it. This epistemological distinction has concrete, formalizable consequences. One consequence is the emergence of "complementarity" for intrinsic observers, even if the underlying system is totally deterministic (computable). Another consequence is the appearence of time and inertial frames for intrinsic observers. The necessary operational techniques are developed in the context of Cellular Automata. We finish with a somewhat speculative question. Given space-time frames generated by clocks which use sound waves for synchronization; why could supersonic travel not cause time paradoxes?

## I. OPERATIONAL CONCEPTS AND INTRINSIC OBSERVERS

John Casti posed the following scenario [7] (for a related discussion, see [15]), "[[Imagine some creatures—]]they may be carbon-based creatures just like you and me. The difference is that they live in a world in which the primary sensory inputs are not from the electromagnetic spectrum like light, but rather come from sound waves. Note that this is not simply a world of the blind; rather, it is a world in which there are no sensory organs for perceiving any part of the electromagnetic spectrum. In this case, then, ... such creatures would see the speed of sound as a fundamental barrier to the velocity of any material object. Yet we as creatures that do possess sensory organs for perceiving the electromagnetic spectrum see the sound barrier as no fundamental barrier at all. So, by analogical extension, there may be creatures "out there" who regard the speed of light as no more of a barrier than we regard the speed of sound."

Casti's scenario is one in which the observers are embedded in a universe which, to them, solely consists of sound waves. Such intrinsic observers develop a description [4, 10, 13–17] of the universe which may appear drastically different (cf. the emergence of complementarity [16, chapter 10]) from what some hypothetical "super-observer" perceives, who is peeking at the system from a "God's eye," extrinsic position. One of the first researchers bothering about these issues has been the 18th century physicist Boskovich, "... And we would have the same impressions if, under conservation of distances, all directions would be rotated by the same angle, ... And even if the distances themselves would be decreased, whereby the angles and the proportions would be conserved, ...: even then we [[the observers]] would have no changes in our impressions. ... A movement, which is common to us [[the observers]] and to the Universe, cannot be observed by us; not

## even if everything would be stretched or shrinked by an arbitrary amount."

Although generally not presented that way, relativity theory consists of two distinct parts: (i) it deals with conventions about how to operationalize certain concepts such as "equal time at spatially separated points," in particularly synchronization procedures; and (ii) it states physical assumptions about the invariance of certain phenomena, such as the speed of light, and the laws of theoretical physics under changes of reference frames. Whereas the former conventions are mere working definitions, the latter statements are supposed to be God-given, eternal symmetries. Conventions can be changed at the price of complicating the theoretical formalism by means of a non-optimal representation of theory. Phenomena are a matter of physical fact. For instance, a preferred frame of reference, e.g., the one at rest with respect to cosmic background radiation, could be artificially introduced. To put it in the words of the late John Bell (cf. [2], p. 34): "You can pretend that whatever inertial frame you have chosen is the ether of the 19th century physicists, and in that frame you can confidently apply the idea of the FitzGerald contraction [[of length of material bodies such as scales]], Larmor [[time]] dilation and Lorentz lag. It is a great pity that students dont understand this. ..."

It was the radical operational feature in Einstein's theory of special relativity, which stimulated the physicist Bridgman. Bridgman demanded that the meaning of theoretical concepts should ultimately be based upon entities which are intrinsically representable and operational. That is, (cf. [6], p. V), "the meaning of one's terms are to be found by an analysis of the operations which one performs in applying the term in concrete situations or in veryfying the truth of statements or in finding the answers to questions." More specifically (cf. [5], p. 103), "... the meaning of length is to be sought in those operations by which the length of physical objects is determined, and the meaning of simultaneity is sought in those physical operations by which it is determined whether two physical events are simultaneous or not."

Therefore, we are free to choose whatever physical concepts seem appropriate as long as they are operational. In particular, Casti's sound-perceiving creatures might choose sound, the author's water-fleas may use water waves, and human physicists may use light for the purpose of Einstein

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synchronization. The resulting frame of references will be different in all these cases. The space-time parameters by which events and phenomena are represented will be different, too. Using different types of formal representations of the same physical system—one may speak of layers or levels of description-might be an effective way to grasp different features of that system (associated, for instance, with different levels of complexity). This resembles Anderson's thesis of "emerging laws" ([1], p. 193; cf. Schweber [11]), "The ability to reduce everything to simple fundamental laws does not imply the ability from these laws and reconstruct the universe. ... The constructionist hypothesis breaks down when confronted with the twin difficulties of scale and complexity. ... at each level of complexity, entirely new properties appear, and the understanding of the new behaviors requires research which I think is as fundamental in nature as any other." It is to be expected that in such an organization of physical concepts, the notion of causality need not be consistently defined for all descriptions combined: what may appear as causal connection (i.e., cause and effect) in one description needs not be causally connected in another description. Therefore it is of great relevance to make precise the limits and applicability of each of these descriptions.

### II. SPACE-TIME FRAMES IN CELLULAR AUTOMATA

In what follows, an explicit example for the construction of space-time frames in computer-generated universes which are generated by *intrinsic* procedures and observations will be given. The onedimensional Cellular Automaton (CA) models [9, 18] considered here are equivalent to any other universal computing agent but have the advantage of good representability on the twodimensional printing page combined with easy programmability.

Our primary concern will be the explicit construction of intrinsic space-time frames by adopting Einstein's synchronization conventions. Stated pointedly, we are interested primarily with "virtual reality" physics and physical epistemology. We do *not* attempt to reconstruct relativity theory one-to-one in the cellular automaton context. In particular, no Lorentzinvariant kinematic theory is introduced. Therefore, certain physical statements, in particular the relativity principle, stating that all laws of physics have an identical form in all inertial frames, needs not to be satisfied (cf. [3]).

However, it has been argued for quite some time [9] that, since all "construable" (in the sense of "recursively enumerable") universes are realisable within the CA framework, also the special theory of relativity can be implemented on such a structure.

#### A. Synchronization

Without loss of generality it is assumed that the maximal velocity, denoted by c, by which a body of information can move is one cell per cycle time. Flows of this kind will be called *rays*. In analogy to relativity theory, this velocity can

I>I0	I>I4	I>_I8
I_>_I0	I_>_I4	I_>_I8
I>IO	I>I4	<u> </u>
I<*0	I<*4	<u> </u>
I_<_I1	I_<_I5	<u> </u>
I <i1< th=""><th>I<i5< th=""><th> <u> </u></th></i5<></th></i1<>	I <i5< th=""><th> <u> </u></th></i5<>	<u> </u>
I>I1	I>I5	I>I9
I_>_I1	I_>_I5	I_>_I9
I>I1	I_>I5	I_>I9
I<*1	I<*5	I<*9
I_<_I2	I_<_I6	I_<_I0
I <i2< th=""><th>I<i6< th=""><th>I<i0< th=""></i0<></th></i6<></th></i2<>	I <i6< th=""><th>I<i0< th=""></i0<></th></i6<>	I <i0< th=""></i0<>
I>I2	I>I6	I>_I0
I_>_I2	I_>_I6	I_>_I0
I>I2	I_>I6	I_>I0
I<*2	I<*6	I<*0
I_<_I3	I_<_I7	I_<_I1
I <i3< th=""><th>I<i7< th=""><th>I&lt;_I1</th></i7<></th></i3<>	I <i7< th=""><th>I&lt;_I1</th></i7<>	I<_I1
I>I3	I>I7	
I_>_I3	I_>_I7	
I>I3	I_>I7	
I<*3	I<*7	
	I_<_I8	
I <i4< th=""><td>I<i8< td=""><td></td></i8<></td></i4<>	I <i8< td=""><td></td></i8<>	

FIG. 1: Ray clock.

be used to define clocks or synchronized events. We shall start by an explicit model of a ray clock. It consists of two mirrors, denoted by I, and a ray of velocity 1 cell per time cycle, denoted by > and <, which is constantly reflected back and forth between the two mirrors, and a one-place digital display right to the right mirror. In this model, after each backreflection of the light ray, the digit on the display increases by one modulus 10. The explicit transformation rules for a onedimensional CA with these properties are listed in appendix A. The time evolution of a ray clock is drawn in Fig. 1.

A very similar configuration as for the light clock can be used as a device for Einstein synchronization [8, 16]: Assume two clocks at two arbitrary points *A* and *B* which are "of similar kind." At some arbitrary *A*-time  $t_A$  a ray goes from *A* to *B*. At *B* it is instantly (without delay) reflected at *B*-time  $t_B$ and reaches *A* again at *A*-time  $t_{A'}$ . The clocks in *A* and *B* are *synchronized* if

$$t_B - t_A = t_{A'} - t_B \quad . \tag{1}$$

The two-ways ray velocity is given by

$$\frac{2|AB|}{t_{A'}-t_A} = c \quad , \tag{2}$$

where |AB| is the distance between A and B.

The ray velocity can then be *defined* to be identical for all frames, irrespective of whether they are moving with respect to the rest frame of the cellular space or not. Of course, this invariance of the ray speed with respect to changes of coordinate systems should ultimately be motivated by phenomenology and a proper choice of conventions. E.g., in relativity theory, the invariance of the Maxwell equations with respect to conformal, or angle preserving, coordinate transformations in four dimensions assures that for light-like vectors,  $ds^2 = dx^2 - (c dt)^2 = 0$  and thus x = ct.

For synchronization, the same CA transformation rules as for the ray clock, which are listed in appendix III can be used. In Fig. 2, an example of synchronization between two clocks A and B is drawn.

clockl	A	в	clock2
I>I0_	_I>	I	_I>I0
I_>_I0_	_I_>	Ι_	_I_>_I0
I_>I0_	_I_>	Ι_	_I>I0
I_<*0_	_I>	Ι_	_I<*0
I_<_I1_	_I>_	Ι_	_I_<_I1
I <i1_< td=""><td>_I&gt;_</td><td>Ι_</td><td>_I<i1< td=""></i1<></td></i1_<>	_I>_	Ι_	_I <i1< td=""></i1<>
I>_I1_	_1>	·I_	_I>I1
I_>_I1_	_I<	*_	_I_>_I1
I_>I1_	_I<	Ι_	_I>I1
I_<*1_	_I<	Ι_	_I<*1
I_<_I2_	_I<	Ι_	_I_<_I2
I <i2_< td=""><td>_I_&lt;</td><td>Ι_</td><td>_I<i2< td=""></i2<></td></i2_<>	_I_<	Ι_	_I <i2< td=""></i2<>
I>I2_	_I_<	Ι_	_I>I2
I_>_I2_	_I<	Ι_	_I_>_I2
I_>I2_	_I>	Ι_	_I>I2
I<*2_	_I_>	Ι_	_I_<*2
I_<_I3_	_I_>	Ι_	_I_<_I3
I<_I3_	_I>	Ι_	_I <i3< td=""></i3<>
I>_I3_	_I>_	Ι_	_I>I3
I_>_I3_	I >	Ι	I > I3
I_>I3_			

FIG. 2: Synchronization by ray exchange.

#### B. Moving reference frames

This section deals with what happens with the intrinsic synchronization and the space-time coordinates when observers are considered which move with respect to the CA medium. For simplicity, assume constant motion of v automaton cells per time cycle. With these units, the ray speed is c = 1, and  $v \le 1$ .

There are numerous ways to simulate sub-ray motion on a CA. In what follows, the case v = 1/3 will be studied in such a way that every three CA time cycles the walls, symbolised by I, move one cell to the right. [Strictly speaking, there should be a periodic transformation of the wall such that  $I \rightarrow a \rightarrow b \rightarrow I$  and the states *a* and *b* have the same reflection properties as I.]

Notice that two clocks which are synchronized in a reference frame which is at rest with respect to the CA medium are *not synchronized* in their own co-moving reference frame. Consider, as an example, the CA drawn in Fig. 3(a). (Strictly speaking, the CA rule here depends on a two-neighbor interaction.) By evaluating equation (1) for  $t_A = 1$ ,  $t_B = 4$ ,  $t_{A'} = 5$ , and  $4 - 1 \neq 5 - 4$ . If the first clock is corrected to make up for the different time of ray flights as in Fig. f:3(b),  $t_A = 2$ ,  $t_B = 4$ ,  $t_{A'} = 6$ , and 4 - 2 = 6 - 4. This correction, however, is the reason for asynchronicity of the two ray clocks with respect to the "original" CA medium.

Let us now explicitly construct the coordinate axes  $\bar{x}$  and  $\bar{t}$  of a system which moves with constant velocity with respect to the medium. For convenience, let us switch to continuous coordinates. That is, the "grainyness" of the CA medium is disregarded; i.e., coordinates will be represented by real numbers. In this construction, the convention of the constancy of the speed of rays for all reference frames will play an important role. The new space and time axes will both become rotated towards the ray coordinates.

Consider Fig. 4, which represents the process drawn in Fig. 3(b). Assume that the two mirrors are *a* (arbitrary) units apart. For the system at rest with respect to the CA medium, a ray is emitted from the origin A = (0,0) and arrives at the mirror at

I>_I0I <ii>_I0</ii>	
I_>IOI_ <i_i_>IO</i_i_>	I_>I1_I_ <i_i_>I0</i_i_>
I_<*0I_ <i_i_<*0< td=""><td>I_&lt;*1I_<ii_<*0< td=""></ii_<*0<></td></i_i_<*0<>	I_<*1I_ <ii_<*0< td=""></ii_<*0<>
I>_I1I>II>_I1	I>_I2I>II>_I1
I_>I1I_>II1_	I_>I2I_>II_>I1
I_<*1I_>I_I_<*1	I_<*2I_>I_I_<*1
I>_I2I_>I_I>_I2	I>_I3_I_>I_I>_I2
I_>I2I>II_>I2	I_>I3I>II2
I_<*2I>II_<*2	I_<*3_I>_I_I_<*2
I>_I3_I>_I_I>_I3	I>_I4I>_II>_I3
I_>I3_I>_I_I_>I3	I_>I4I>_II_>I3
I_<*3_I>I_I_<*3	I_<*4I>II_<*3
I>_I4I>II>_I4	I>_I5_I>II>_I4
I_>I4I<*I_>I4	I_>I5I<*I_>I4
I_<*4_I <i_i_<*4< td=""><td>I_&lt;*5_I&lt;_I_I_&lt;*4</td></i_i_<*4<>	I_<*5_I<_I_I_<*4
I>_I5I <i_i>_I5</i_i>	I>_I6_I_ <i_i>_I5</i_i>
I_>I5I_ <ii>I5</ii>	I_>I6I_ <ii>I5</ii>
I_<*5I_ <ii_<*5< td=""><td>I_&lt;*6I_<ii_<*5< td=""></ii_<*5<></td></ii_<*5<>	I_<*6I_ <ii_<*5< td=""></ii_<*5<>
I>_I6I>II>_I6	I>_I7I>II>_I6
I_>I6I_>II_>I6	I_>I7I_>I>I6
I_<*6I_>I_I_<*6	I_<*7I_>I<*6
I>_I7I_>I_I>_I7	I>_I8_I_>I_I>_I7
I_>I7I>II7	I_>I8I>II_>I7
I_<*7I>II_<*7	I_<*8I>II_<*7
I>_I8II_I>_I8	I>_I9_I>_I_I>_I8
I_>I8I>_II_>I8	I_>I9I>_II_>I8
I_<*8I>II_<*8	I_<*9I>II_<*8
I>_I9I>II>_I9	I>_I0I>II>_I9
I_>I9I<*_I_>I9	I_>I0I<*I_>I9
I_<*9I<_II_<*9	I_<*0I<_II_<*9
I>_I0I_ <i_i>_I0</i_i>	I>_I1I <i>_I0</i>
I_>I0I_ <i_>I0</i_>	I_>I1_I_ <i_>I0</i_>
I_<*0I_ <i_i_<*0< td=""><td>I_&lt;*1I_<i<*0< td=""></i<*0<></td></i_i_<*0<>	I_<*1I_ <i<*0< td=""></i<*0<>
I>_I1I>II>I1	I>_I2I>II>_I1_
I_>I1_I_>I_>I1_	I_>I2I_>I_>I1_
I_<*1I_>I_<*1	I_<*2I_>II_<*1_
I>_I2_I_>I_I>_I2_	I>_I3_I_>I_I>_I2
I_>I2I_>II2	I_>I3_I_>I_I2
I_<*2I>II_<*2	I_<*3_I>_I_I_<*2
I>_I3_I>_I_I>_I3_	I>_I4I>_I_I>_I
I_>I3I_>_II3	I_>I4I>_I_I>I
I_<*3_I>II_<*3	I_<*4I>II_<*
I>_I4I>II>_I4	
I_>I4I_<*I_>I4	
I_<*4I<_II_<*4	I_<*5I<_II_<
(a)	(b)

FIG. 3: Unsynchronized (a) and synchronized (b) ray clocks in moving reference frame.

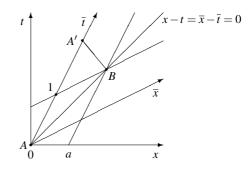


FIG. 4: Construction of the coordinate axes  $\bar{x}$  and  $\bar{t}$  of a system which moves with constant velocity with respect to the medium.

 $B = (vt_B + a, t_B)$ , where it is reflected and arrives at the original mirror at  $A' = (vt_{A'}, t_{A'})$ . Since *B* lies on the ray which comes from the origin,  $B = (t_B, t_B) = (\frac{a}{1-v}, \frac{a}{1-v})$ . Since *A'* lies on the ray which comes from *B*,  $t_{A'} = -x_{A'} + C$  and  $t_B = -x_B + C$ . By evaluating *C*, one obtains  $A' = (\frac{2va}{1-v^2}, \frac{2a}{1-v^2})$ . In the coordinate system which is moving with respect to the CA medium,  $\overline{A} = (0,0)$ . In order for the clocks to be synchronized, i.e., by equation (1),  $\overline{t}_B = (\overline{t}_{A'} + \overline{t}_A)/2$ . But this should also be the time coordinate  $\overline{t}_1$ , since this is just half the time from *A* to *A'*. Thus one obtains two points (events)  $1 = (\frac{vt_{A'}}{2}, \frac{t_{A'}}{2})$  and *B* whose time coordinates in the moving reference frame is identical; i.e.,  $\overline{t}_1 = \overline{t}_B$ . A short calculation shows that, with respect

to the coordinate system which is at rest in the CA medium, the lines of equal time coordinates for a system wich is moving with constant velocity v, e.g., the  $\bar{x}$ -axis, have slope 1/v, whereas the lines of equal space coordinates, e.g., the  $\bar{t}$ -axis, have slope v.

These transformation of coordinate axes correspond to the Lorentz transformations

$$\bar{t} = \frac{t - vx}{\sqrt{1 - v^2}}$$
,  $\bar{x} = \frac{x - vt}{\sqrt{1 - v^2}}$ . (3)

The specific form of the transformation (3) comes as no surprise, since it has been derived by implicitly assuming that constant motion transforms into constant motion and that the ray speed is the same for all reference frames; both conditions being the kinematic equivalent to the relativity principle.

#### C. Sub-ray synchronization

So far, only rays propagating one CA cell per cycle have been considered. It is not entirely unreasonable to assume synchronization with signals which propagate slower (or faster) than these rays.

A typical example would be the use of a signal for synchronization with slower-than-ray speed, i.e., c' < c.

One consequence of sub-ray synchronization is the possibility of "super-ray" speeds v such that  $c \ge v > c'$ , and of a "time travel" with respect to such space-time frames. That is, for certain observers moving with v > c', the time coordinate defined by c' would "run backward." This "time travel" however, is nothing particularly mysterious, but the outcome of the specific synchronization convention chosen (cf. below).

# III. CAN SUPERSONIC TRAVEL GIVE RISE TO TIME PARADOXES?

Let us come back to Casti's sound-sensitive creaturessuppose that they call themselves SMORONS. Suppose further that they discover the possibility to generate and detect light; e.g., by sonoluminescence. It can be expected that this discovery will cause a major trauma for the SMORONS, because they will find out that they could communicate much faster than by sound waves, on which their space-time frames are based. If they have applied the Einstein conventions for defining space-time frames, they would observe light as a supersonic (i.e., faster-than-sound) signal, which propagates on space-like  $((\Delta x)^2 - (\Delta t)^2 > 0)$  world lines. Such a phenomenon which might allow forward in time signalling in one inertial frame could allow backward in time signalling in another inertial frame; there always exists some (orthochronous) Lorentz transformation L which transforms a space-like wordline with t > 0 into one with t < 0.

Given backward in time signalling, a classical time paradox can be formulated: Assume two observers A and B which can communicate *backward* in time; i.e., a signal traverses the distance  $x_{AB}$  between them in time  $t_{AB}$  such that  $t_{AB} < 0$ . Then the observer A might emit a signal at time  $t_A$ , which arrives the observer *B* in  $t_B$ , where it is reflected and is back at the observer *A* at a time  $t_{A'} < t_A$ , i.e., *before* observer *A* has emitted the original signal. If one performs a "diagonalization," i.e., if one assumes that observer *A* emits a signal at time  $t_A$  if and only if *no* signal is absorbed at  $t_{A'}$ ; observer *A* emits *no* signal at time  $t_A$  if and only if a signal is absorbed at  $t_{A'}$ , one ends up with the simplest form of time paradox.

The syntactic structure of this paradox closely resembles Cantor's diagonalization method (based on the ancient liar paradox), which has been applied by Gödel, Turing and others for undecidability proofs in a recursion theoretic setup.

How could the SMORONS cope with such a time paradox?

One could argue that sound consists of elementary constituents (such as atoms or molecules or clusters thereof), whose motion is ultimately governed by electromagnetic forces. Therefore, the valid theory is electromagnetism, and any theory "shell" ("level") such as the SMORON theory of sound waves must ultimately be based upon (although not necessarily be totally derivable from) it. As has been pointed out earlier, such a "shell" is very similar to what Anderson [1] calls "emerging law" (cf. Schweber [11]). In this picture, the elementary constituents are a sort of "hidden parameters" for SMORON physics; something they can neither observe nor control. Therefore, the SMORON physics does not apply to configurations in which their physics "shell" is inappropriate. They are unable to control the events. This is why the theory is cryptodeterministic, and the SMORONS have no free will with respect to diagonalization-they will simply not be able to operationalize diagonalization purely in terms of sound waves.

One major goal of these considerations is the assertion that space and time are not God-given, metaphysical objects, but are subject to theoretical *construction*. The construction of space-time depends on *conventions*. Any inconsistency, for instance the possibility to construct time paradoxes, may be perceived as a problem of the improper, unfaithful construction of space and time rather than the impossibility to do certain tasks such as super-fast signalling. Any unfaithful construction of space-time may in turn be deeply rooted in the status quo of physical theory.

## APPENDIX A: TRANSFORMATION RULES OF A CA LIGHT CLOCK

$$\begin{split} & \varphi(>, .., X) =>, \varphi(X, .., <) =<, \varphi(-, .., -) = ., \varphi(X, .., >) = ., \\ & \varphi(<, .., X) = ., \varphi(-, >, .) = ., \varphi(-, <, .) = ., \varphi(-, >, I) =<, \\ & \varphi(I, <, .) =>, \varphi(>, I, X) = *, \varphi(<, *, X) = I, \varphi(X, <, *) = ., \\ & \varphi(*, 1, X) = 2, \varphi(*, 2, X) = 3, \varphi(*, 3, X) = 4, \varphi(*, 4, X) = 5, \\ & \varphi(*, 5, X) = 6, \varphi(*, 6, X) = 7, \varphi(*, 7, X) = 8, \varphi(*, 8, X) = 9, \\ & \varphi(*, 9, X) = 0, \varphi(*, 0, X) = 1, \varphi(0, .., X) = ., \varphi(1, .., X) = ., \\ & \varphi(2, .., X) = ., \varphi(3, .., X) = ., \varphi(4, .., X) = ., \varphi(5, .., X) = ., \\ & \varphi(6, .., X) = ., \varphi(7, .., X) = ., \varphi(4, .., X) = ., \varphi(5, .., X) = ., \\ & \varphi(K, *, 0) = *, \varphi(X, *, 1) = *, \varphi(X, *, 2) = *, \varphi(X, *, 3) = *, \\ & \varphi(X, *, 8) = *, \varphi(X, *, 9) = *, \varphi(X, 1, X) = 1, \varphi(X, 2, X) = 2, \\ & \varphi(X, 3, X) = 3, \varphi(X, 4, X) = 4, \varphi(X, 5, X) = 5, \varphi(X, 6, X) = 6, \\ & \varphi(X, 7, X) = T, \varphi(X, 8, X) = 8, \varphi(X, 9, X) = 9, \varphi(X, 0, X) = 0, \\ & \varphi(X, 1, 0) = I, \varphi(X, I, 1) = I, \varphi(X, I, 2) = I, \varphi(X, I, 3) = I, \\ \end{aligned}$$

- $,\mathtt{I})=_{-}\quad .$ 
  - X stands for any state.
- [1] P. W. Anderson, *More is Different*, *Science* **177**, 393 (August 1972).
- [2] J. Bell, *George Francis FitzGerald*, Physics World **5**, 31-35 (1992).
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