ARTICLES

Interaction-free preparation

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We demonstrate that the preparation of a very well localized atom beam is possible without physical interaction. The preparation is based on the selection of an adequate ensemble of atoms of an originally wide beam by means of information obtained with a neutron interferometer. In such a case, the uncertainty relation can no longer be interpreted as a by-product of the interaction between the system and the preparation apparatus.

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INTRODUCTION

In 1927 Heisenberg formulated the uncertainty relation, which expresses the fact that the expectation values of two noncommuting observables cannot be determined with arbitrary precision. He demonstrated this by means of a γ -ray microscope, which since then has been discussed in many textbooks of quantum mechanics. In such a (gedanken) microscope the location of an electron is determined by γ -ray photons that are scattered on the electron. Due to the Compton effect, the momentum of the electron will be changed when the position measurement (scattering of the photon) takes place. Because the resolution of the position measurement is related to the wavelength of the photons, the momentum transfer in the scattering process will increase as the accuracy of the position measurement is increased. Therefore it is not possible to determine both position and momentum with arbitrary precision.

This and many other examples that have been invented to illustrate the meaning of the uncertainty relation may lead to the assumption that this relation is always based on a physical interaction between the measured system (electron) and the system by which the measurement is performed (photon). This assumption is reasonable when it is assumed that no measurement is possible without physical interaction. (Cf. [1]: measurement by interaction is associated with the exchange of at least one quantum of action.) The term "physical interaction" is used here for processes that are associated with the exchange of at least one quantum of action.

The same considerations may also be applied to the preparation process. In experiments, properties such as the spatial extension or the energy of a system usually are controlled by methods that imply a physical interaction with the system. Thus the limits in defining the initial conditions of a system as expressed by the uncertainty relation may again be interpreted as a consequence of the physical interaction occurring in the preparation process.

In this paper we will discuss a preparation method that involves no physical interaction, thereby strictly excluding such a mechanistic interpretation of the uncertainty relation. In the proposed setup we use the idea of interaction-free measurement, which has been presented by Elitzur and Vaidman [2-4]. They have shown that the presence of an object can be detected without interacting with the object by making use of a Mach-Zehnder interferometer. This interactionfree measurement scheme has been optimized and realized in an experiment performed by Kwiat *et al.* [5,6].

THE GEDANKEN EXPERIMENT

In its simplest form, an interaction-free measurement can be made with a Mach-Zehnder interferometer (cf. Fig. 1). When this kind of interferometer is empty, the amplitudes leading to detector D_2 interfere destructively and therefore only detector D_1 can fire. If we insert into path I an object



FIG. 1. Mach-Zehnder interferometer. S denotes the source and D_1 and D_2 are detectors. If there is no absorber (opaque object) present in path I, output 2 is dark and only detector D_1 fires. As soon as path I is blocked, both detectors can fire. In case D_2 fires one knows that an absorber is present in path I without having interacted with it.

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FIG. 2. Neutrons from the source NS are incident on a Mach-Zehnder neutron interferometer and can finally be registered by detectors D_1 and D_2 . In path I the neutron beam propagating along the *x* direction is crossed by a ¹⁵⁷Gd atom beam that is parallel to the *z* direction. It is assumed that the atom beam is wider than the neutron beam, as shown in detail in the inset.

that is assumed to be a perfect absorber, this path is blocked and no interference can occur. Then both detectors will fire with equal probability. Thus, if a single photon is sent into the interferometer and a click is detected in D_2 , one knows with certainty that an object is present in path I without having interacted with this object. Of course it is also possible that the photon is absorbed by the object or detected in D_1 , but nevertheless in 25% of all trials we will succeed in performing an interaction-free measurement. With a more complicated setup the percentage of successful trials can come arbitrarily close to 100% [5].

We now turn to our method of interaction-free preparation of a narrow atom beam from an originally wide one. Consider the Mach-Zehnder interferometer for neutrons shown in Fig. 2. Let w_n be the width of the beams inside the interferometer. In path I the neutron beam propagating along the xdirection is crossed by a beam of ¹⁵⁷Gd atoms that is parallel to the z direction and has the width w_{Gd} (cf. Fig. 2). We use ¹⁵⁷Gd atoms because they are highly efficient neutron absorbers. The Gd beam is assumed to be wider than the neutron beam $(w_{\text{Gd}} \ge w_n)$. Without the atom beam all neutrons passing through the interferometer will be detected in D_1 . As soon as we turn on the Gd beam, path I of the neutron interferometer will be blocked once in a while by an atom, which acts as a neutron absorber. Then also detector D_2 can fire. If it fires one knows that the Gd atom was present within the region of width w_n defined by the neutron beam in path I. Because path I was blocked by the Gd atom one also knows that the neutron detected in D_2 took path II and therefore never interacted with the Gd atom. Interaction-free preparation of a Gd beam of width w_n from an originally much wider beam is thus possible by installing a shutter for the Gd beam after the overlap with the neutron beam in path I. This shutter opens only— with a suitable time delay — when a neutron is detected in D_2 , thereby permitting the selected Gd atom to pass on.

FORMAL DESCRIPTION

We now turn to a more detailed discussion. For the sake of simplicity the neutron beam is assumed to be of rectangular cross section with constant transverse probability density. This comes close to real experimental conditions. An analogous assumption is made for the atom beam. For the following it is sufficient to consider only one transverse direction of the beams. Similarly, their longitudinal description can be ignored. Because $w_{Gd} \ge w_n$ is assumed, we represent the transverse probability density of the atom beam in real space as a superposition of a rectangular wave packet $|a\rangle_R$ of width w_n , which exactly crosses the neutron beam (cf. Fig. 2), and of another wave packet $|a\rangle_0$, which represents the rest of the beam:

$$|a\rangle = |a\rangle_R + |a\rangle_0$$
.

When the neutron and atom wave packets overlap the following processes can happen.

- (i) The neutron and atom do not interact.
- (ii) The neutron is scattered by the atom.
- (iii) The neutron is absorbed by the atom.

Corresponding to these possibilities the combined state of the atom and of the neutron in path I after the overlap is given by

$$|n\rangle_{\mathrm{I}}|a\rangle = |n\rangle_{\mathrm{I}}(|a\rangle_{0} + c|a\rangle_{R}) + \sum_{l} s_{l}|n\rangle_{\mathrm{I},l}|a\rangle_{R,l} + z|n\rangle_{\mathrm{I}}|a\rangle_{R}.$$
(1)

Here s_l are the probability amplitudes for scattering, where l labels the exchange of momentum and kinetic energy between neutron and atom, and therefore also appears in the resulting state vectors. s_0 is the amplitude for forward scattering, which changes neither the state of the neutron nor that of the atom, but adds a phase factor. The absorption amplitude is given by z. The amplitude c in the first term on the right-hand side expresses the probability that the Gd atom crosses through the neutron beam without scattering or absorption and is given by

$$c = \sqrt{1 - \sum_{l} |s_{l}|^{2} - |z|^{2}}$$

Note that for the interaction of slow neutrons with ¹⁵⁷Gd, scattering is four orders of magnitude less likely than absorption because the cross section for absorption is 2.5×10^5 b (10^{-24}cm^2) , whereas that for scattering is of the order of 10 b. (No exact value is known for ¹⁵⁷Gd. The value for natural gadolinium, which contains 15.65% of ¹⁵⁷Gd, is 7 b.) Consequently, we have $\Sigma_l |s_l|^2 \ll |z|^2$.

Now the probability amplitude for detection of a neutron in D_2 can be calculated. The neutron can reach detector D_2 by the following routes. (i) It passes through the interferometer along path II.

(ii) It passes through the interferometer along path I and is neither scattered (except forward scattering) nor absorbed.

Thus we get, for the combined state of the neutron just before detector D_2 and of the atom,

$$|n\rangle_{D_2}|a\rangle = |n\rangle_{\mathrm{II},D_2}|a\rangle + |n\rangle_{\mathrm{I},D_2}|a\rangle_0$$
$$+ c|n\rangle_{\mathrm{I},D_2}|a\rangle_R + s_0|n\rangle_{\mathrm{I},D_2}|a\rangle_R.$$
(2)

The two states contributing to the output of the Mach-Zehnder interferometer towards detector D_2 are functions of the input state $|n\rangle_0$. Neglecting the directions of the beams, these states are given by

$$|n\rangle_{\mathrm{I},D_{2}} = \frac{i}{2}|n\rangle_{0},$$
$$|n\rangle_{\mathrm{II},D_{2}} = -\frac{i}{2}|n\rangle_{0},$$
(3)

such that Eq. (2) can be rewritten as

$$|n\rangle_{D_2}|a\rangle = \frac{i}{2}(c+s_0-1)|n\rangle_0|a\rangle_R.$$
 (4)

With realistic dimensions of the beam width w_n , from a few micrometers upward, and with the usual very sparse beams, most of the time the neutrons and the Gd atoms will not interact ($c \approx 1$). But in the rare cases when an interaction occurs it is predominantly absorption because of $|z|^2 \gg \Sigma_l |s_l|^2$ and $|z|^2 \gg |s_0|$. Therefore Eq. (4) reduces to

$$|n\rangle_{D_2}|a\rangle \approx -\frac{i}{4}|z|^2|n\rangle_0|a\rangle_R.$$
(5)

Equation (5) expresses the fact that by detecting a neutron in D_2 one has reduced the original state of the atom $|a\rangle = |a\rangle_R + |a\rangle_0$ to $|a\rangle_R$. The atom is thus indeed confined to a wave packet, that has the width of the neutron beam. This corresponds to a gain of knowledge about the position of the atom. Because the neutron by which this gain has been reached almost always took path II and therefore could not have interacted with the atom, this is a method of preparing the state $|a\rangle_R$ without any physical interaction.

DISCUSSION

Our analysis has shown that information about the presence of an atom can apparently be obtained without interaction and can be used for preparing an atomic beam as narrow as the "probing" neutron beam. In view of the original treatment of an interaction free measurement by Dicke [7], which concluded that the absence of interaction was only an illusion, it is worthwhile to investigate in what sense there was no interaction between the neutrons and those atoms, which ultimately compose the narrow beam.

Dicke considered the Gaussian wave packet of an atom traversed by a beam of light much narrower than the wave packet. Photons scattered at the atom are detected, whereby one learns that the atom was within the beam of light. If no scattered photons are detected with a properly adjusted intensity of the light, one learns that the atom is outside the beam of light. This information seems to have been gained without interaction. It yields a new wave function showing the atom localized somewhere in a ring around the beam of light. This is a narrower structure than the atom's original wave packet, with a corresponding increase of the kinetic energy. Where could this additional energy have come from? By means of perturbation theory Dicke ascribes this to the absorption and reemission of a photon by the atom. The reemitted photon is not detected as scattered because it is within the momentum uncertainty of the focused beam of light. Thus the measurement result "atom is outside the beam of light" is only apparently obtained without interaction.

There are two essential differences between the measurement scheme discussed by Dicke and the preparation method presented here. One is that in our setup information about the system is gained by means of interference. The other is that in addition to scattering we also consider absorption. Nevertheless, all effects discussed by Dicke are relevant in order to describe what happens between the atoms and neutron beam I.

If there were only neutron beam I, we could observe the absorption of a neutron by an atom by detecting the highenergy photon emitted by the atom in the transformation processes of the nucleus or we could detect the scattered neutron. Both processes would correspond to the detection of a photon scattered by the atom in the case discussed by Dicke. If the neutron is not absorbed by an atom, it did not "see" the atom or it was scattered in forward direction accounted for by the amplitude s_0 in Eq. (1). Forward scattering is the interaction analogous to scattering within the momentum uncertainty of the beam of light in Dicke's case.

Thus, without an interference loop for the neutron, information about the localization of the atom would be obtained in a similar way as in Dicke's case: Detection of a highenergy photon or of the scattered neutron would indicate that the atom was within the width of the neutron beam. If neither effect is present, we obtain a new wave function for the atom, which has a smaller amplitude in the region crossed by the neutron beam.

With the interference loop, however, the absence of a high-energy photon or a scattered neutron may result in *two* different informations, as either D_1 or D_2 may fire. The firing of D_1 tells us little about the new wave function for the atom. But when D_2 fires we can think of two different causes.

(i) It may be due to the phase shift the neutron acquired in forward scattering at the atom, and hence due to an interaction in the region of the neutron beam. This localizes the atom within the width of neutron beam I. Naturally, it cannot be said whether the interaction has actually taken place, as path II is also open to the neutron.

(ii) Alternatively it may be due to the atom acting as an absorber and blocking path I. But rather than being really absorbed, the neutron took the other path available in the interferometer. This too localizes the atom within the width of neutron beam I, which could be interpreted as gaining information by "frustrated absorption." For the parameters

in our example this is by far the main reason for a firing of D_2 .

In this context two facts are important. First, it should be noted that in Dicke's case nondetection of a scattered photon localizes the atom *outside* the beam of light, whereas in our case the analogous processes localize the atom *within* the neutron beam. Second, the possible absorption of the neutron does remain an *unused* possibility because otherwise one would have detected a high-energy photon. One can only conclude that these neutrons have come along path II and hence have not interacted with the atoms. In Dicke's case nothing analogous can be found.

The narrow beam of selected atoms must fulfill the Heisenberg uncertainty relations, which require the trans-

verse kinetic energy to have increased. For most of the atoms this cannot have happened as they are selected by frustrated absorption, where really no interaction seems to occur. Only the contribution from forward scattering leaves room for the exchange of energy from the neutron to the atom. This can indeed account for the necessary change of energy because according to the optical theorem, the total reaction cross section, which in our case includes scattering and absorption, is proportional to the amplitude for forward scattering.

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