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Multiple-channel fractal information coding of mammalian nerve signals

K. Svozil, *D. Felix,+ and K. Ehrenberger#

*Institut für Theoretische Physik, Technische Universität Wien
Wiedner Hauptstraße 8-10/136, A-1040 Vienna, Austria

+Department of Neurobiology, University of Berne
Erlachstraße 9a, CH-3012 Berne, Switzerland

#ENT Department, University of Vienna
Währinger Gürtel 18-20, A-1090 Vienna, Austria

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As an average of minimal 3 to maximal 30 single auditory-nerve fibers converge in the auditory pathway, the fractal geometry of their signals is transformed to a different fractal geometry such that small variations of the primary discharge patterns correspond to large variations of the combined signal. The addition of white noise does not affect the fractal signal structure. The quality of the transsynaptic information transfer depends on the relation between the number of the convergent spike trains and the individual fractal geometries of the convergent spike trains.

$1/f$-spectra and, related to them, fractal dynamics in nervous systems, have been extensively discussed in the literature [1, 2, 3, 4, 5]. Spike discharge patterns can be represented as point sets or sequences of symbols, most simply, of binary symbols. In cochlear afferents, acoustically driven spike trains exhibit an irregular and bursting behaviour corresponding to random fractal structures [6, 7]. The iontophoretic release of transmitter agonists in the synaptic cleft of inner hair cells (IHC) triggers equivalent bursting discharges [8, 9]. In the auditory system, these primary fractal geometries are processed by diverging and converging information networks. In mammals, one cochlear IHC is innervated by minimal 3 up to maximal 30 afferents [10]. The corresponding, parallely propagated information flow converges in succeeded structures of the auditory pathway [11]. The aim of this paper is to model the functional conditions of a reliable flow of fractally coded information under these structural conditions, in particular for converging parallel channels and to estimate the matched fractal dimension of this information flow.

It is assumed that the convergence in the auditory pathway can be modeled by an integrator device which "fires" only if the cumulative impulse from the single auditory-nerve fibers is strong enough; more specifically, if there is an input from *all* single auditory-nerve fibers. Stated differ-

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ently: this model is based on the assumption that the secondary signal is the *intersection of inputs* from all single auditory-nerve fibers. This amounts to the study of intersection of fractal sets.

In the following n random fractal signals, denoted by A_i, $1 \leq i \leq n$, are represented by sequences of zeros and ones. Each one of these sequences is transmitted in a separate channel. The sequences are then recombined to form a new, secondary signal sequence. This setup is drawn in Fig. 1. As has been stated earlier, the formation rule of the secondary signal sequence is equivalent to the set theoretical intersection. I.e., the i'th place of the secondary signal sequence is 1 if and only if all the i'th places of the incoming sequences are 1.

As has been pointed out by K. J. Falconer [3] and as can be readily verified by computer experiments, under certain "mild side conditions," the dimension $D^{\cap}$ of the intersection of n random fractals A_i, $1 \leq i \leq n$, can be approximated by

$$D^{\cap}(\{A_i\}) = D(\bigcap_{i=1}^{n} A_i) \approx \sum_{i=1}^{n} D(A_i) - n + 1 \quad . \tag{1}$$

Consider the coding of a signal by random fractals by their *dimension* parameter. An example is the case of just two source symbols s_1 and s_2 encoded by (*RFP* stands for "random fractal pattern")

$$\#(s_i) = \begin{cases} RFP \quad \text{with} \quad 0 \leq D(RFP) < 0.5 \quad \text{if} \quad s_i = s_1 \\ RFP \quad \text{with} \quad 0.5 \leq D(RFP) \leq 1 \quad \text{if} \quad s_i = s_2 \end{cases} \quad . \tag{2}$$

Let us call the A_i's the *primary signals* (primary sources), and the intersection $\bigcap_{i=1}^{n} A_i$ of the primary signals as the *secondary signal* (secondary source). We shall study interesting special cases of equation (1). The addition of white noise to a random fractal signal, denoted by $\mathbb{I}$ with $D(\mathbb{I}) \approx 1$, results in the recovery of the original fractal signal with the original dimension; i.e.,

$$D^{\cap}(A, \mathbb{I}) \approx D(A) + D(\mathbb{I}) - 1 \approx D(A) \quad . \tag{3}$$

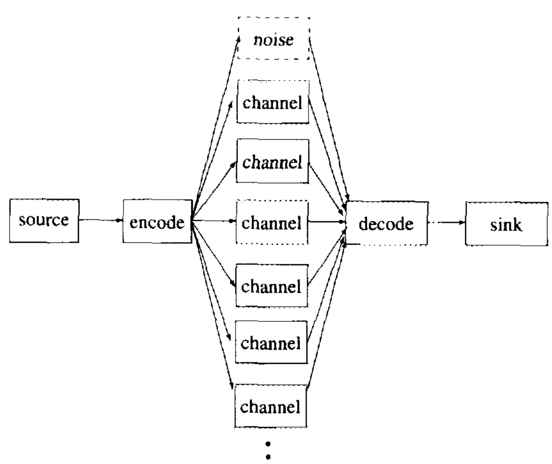

Figure 1. Signal transmission *via* multiple channels.

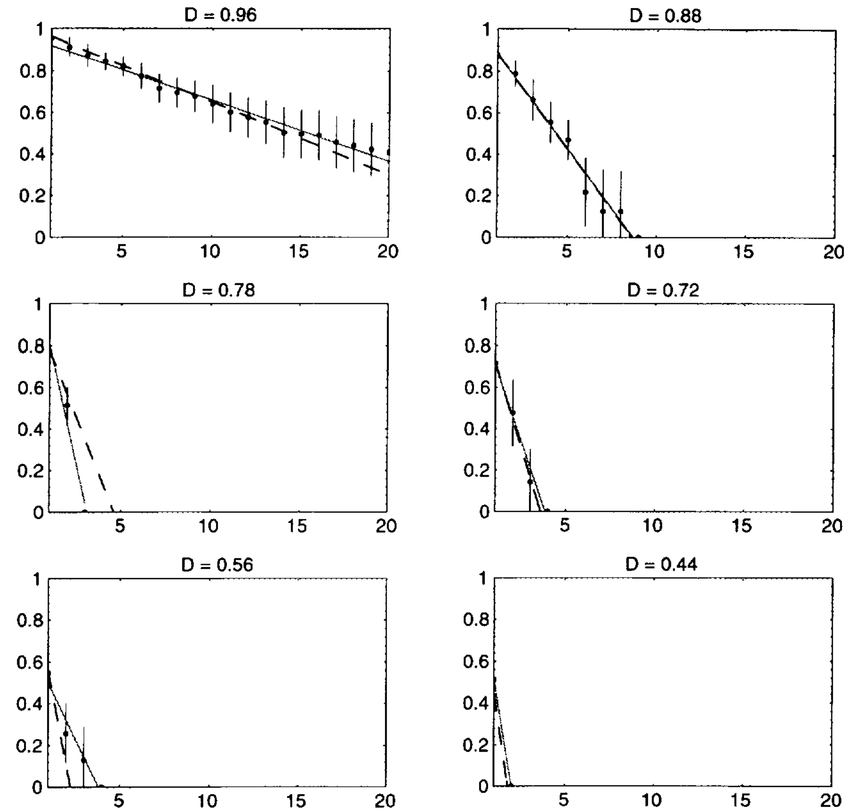

Figure 2. $D^{\cap}(n)$ versus n for various values of dimensions D. The points indicate data points generated from computer experiments. The solid grey line indicates the quadratic fit to a linear function. The dashed line indicates the theoretical prediction.

By assuming that all random fractals have equal dimensions, i.e., $D(A_i) = D$ and $D^{\cap} \geq 0$, equation (1) reduces to

$$D^{\cap} \approx \max\{0, n(D-1)+1\} \tag{4}$$

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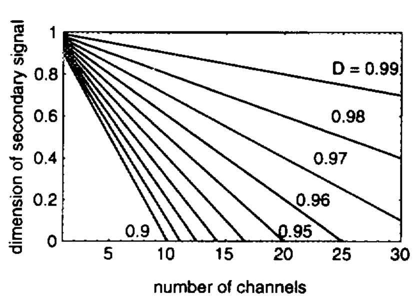

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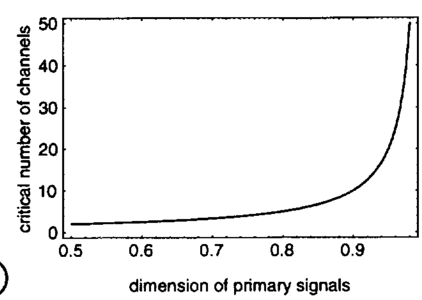

Figure 3. Theoretical prediction of $D^{\cap}(n)$ versus n for various values of dimensions D.

Figure 4. Theoretical prediction of the critical number of channels as a function of the dimension of the primary signal.

In Fig. 2, the dimension $D^{\cap}(n)$ of the secondary signal is drawn as a function of the number of channels $1 \leq n \leq 20$ for various values of the dimension D of the primary signal. The primary signals were generated numerically; the secondary signals were measured by box-counting methods. Fig. 3 shows the theoretical prediction of $D^{\cap}(n)$ versus n for various values of dimensions D. From these figures it can be seen that, intuitively speaking, the larger the number of channels, the higher the dimension of the primary signal has to be in order to obtain a secondary signal of nonzero dimension. Fig. 4 shows the theoretical prediction of the critical number of channels as a function of the dimension of the primary signal.

An immediate consequence of (4) is that, for truely fractal signals ($D < 1$), any variation of the fractal dimension of the secondary signal $D^{\cap}$ is directly proportional to the number n of the primary signals; i.e.,

$$\Delta D^{\cap} \approx n \Delta D \quad \text{for} \quad D \neq 1 \quad . \tag{5}$$

In consequence, the more channels there are, the more the dimension of the secondary source varies in response to variations of the primary source; there is an "amplification" of any change in the primary signal. This amplification augments the signal-to-noise ratio and therefore contrasts the relevant information in the sequence of discharges, corresponding to a third, neuronal filter tandem-arranged to the first two mechanical filters of the cochlea [12]. This third filter is not disturbed by concommitant white noise with the dimension $D = 1$, because in this case, $dD^{\cap}/dD = 0$, there is no such "amplification" and the effect vanishes. Therefore, this neuronal filter not only contrasts but stabilizes the fractally coded information flow.

Between the amplification and the strength of the fractally coded signals a tradeoff exists: any increase in the amplification of the variation of the primary dimension obtained by additional channels results in a reduction of the overall secondary signal activity. For a channel number of $10 - 20$, the fractal dimension of the primary signal has to be within the $0.9 - 1$-range in order to balance this weakening of the signal.

Fractal signals with lower dimensions must be processed by networks with a lower number of active converging channels. Therefore in neuronal networks, arranged in divergent-convergent circuits, the quality of the information transfer depends on an optimal relation between the number of convergent channels and the dimension of the fractally coded primary signals.

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