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Brief Reports

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Test of local causality with very short light pulses

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It is explicitly shown that the nonvanishing spacelike or timelike contributions to the Feynman propagator of quantum-field theory do *not* reflect any violation of local causality, such as propagation faster than light. An experiment is proposed to test the causality of quantum-field theory by measuring the broadening of a very short light pulse. For these pulses theory predicts a nondissipative behavior, with the pulse broadening only stemming from preparation and measurement uncertainties.

The usual implementation of local causality in quantum-field theory requires independence of the field amplitudes at spatially separated points,[1] since according to theory of relativity no event can be caused from regions outside of its own light cone. To distinguish this restricted causality from earlier ones (which allow for instantanuous action at a distance, such as Newton's law of gravitation) it shall be called "local" (also known as micro- or Einstein) causality. Local causality is assured by a proper connection between spin and statistics; e.g. for the case of a (massive) scalar field ϕ, the commutator is given by the Pauli-Jordan function

$$[\phi(0),\phi(x)]=i[\Delta_R(x)-\Delta_A(x)]$$
$$=\frac{i}{4\pi(t^2-\mathbf{x}^2)^{1/2}}\{\delta(t-|\mathbf{x}|)-\delta(t+|\mathbf{x}|)$$
$$-\epsilon(t)mJ_1[m(t^2-\mathbf{x}^2)^{1/2}]\}\ ,$$

where Δ_R and Δ_A denote the retarded and advanced Green's functions, respectively. The commutator vanishes for spacelike separations $t^2-\mathbf{x}^2<0$, that is outside the light cone. This presupposes the invariance of suitable velocities corresponding to propagation processes used for synchronization. Since gravitational waves are presently unattainable, for all practical purposes, the speed of light or other electromagnetic radiation have been defined as a unique value, $c=229\,792\,458$ m/sec (the units throughout this paper will be such that $c=1$ and $\hbar=1$).

It is, however, not evident that this uniqueness of the velocity of light (corresponding to a singular distribution) holds for quantized systems as well. One might even attempt to consider the velocity of light (which is inserted into the "bare" theory) as a parameter which becomes renormalized *en route* to the full model, very much like mass or charge. Moreover, via the Wick decomposition, perturbative-quantum-field theory induces a definition of the Green's function with nonvanishing contributions for spacelike separated points: for massive fields the "causal" Green's function in configuration space is

$$\Delta_c(x)=\langle 0|T\phi(0)\phi(x)|0\rangle$$
$$\propto\theta(-x^2)[m/(-x^2)^{1/2}]K_1[m(-x^2)^{1/2}]+\cdots\ ,$$

which for the massless case ($m=0$) and for small $x^2=t^2-\mathbf{x}^2\neq 0$ (close to the light cone) can be expanded, yielding $\Delta_c(x)\propto x^{-2}$.

The discussion of possible causality violations due to the specific form of the causal Green's functions dates back to the early days of quantum theory.[2] Fierz[3] and Källen[4] have attempted to argue via the uncertainty principle. This argument shall be briefly reviewed here, since on closer inspection it seems unconvincing. They attempt to prove that although quantum-mechanical scattering amplitudes in principle show nonlocal contributions (as well as others indicating slower propagation than c), due to the uncertainty principle these cannot be detected. They start by tesselating space-time into disjoined regions R_i such that the (second-order) amplitude for Møller scattering of a photon is a sum

$$\langle p_1'p_2'|S^{(2)}|p_1p_2\rangle=\sum_{R_1,R_2}S^{(2)}(R_1,R_2)$$

of all scattering processes from one region R_1 into another R_2. $S^{(2)}(R_1,R_2)$ can be evaluated as $[D_c^{\mu\nu}(x)\propto\Delta_c(x)]$

$$\frac{e^2}{2}\int_{R_1} d^4x \int_{R_2} d^4y \langle p_1'|\bar{\psi}(x)\gamma_\mu\psi(x)|p_1\rangle D_c^{\mu\nu}(x-y)\langle p_2'|\bar{\psi}(y)\gamma_\nu\psi(y)|p_2\rangle + \cdots$$

$$\propto \int_{-\infty}^{+\infty} \frac{\delta_{R_1}^{(4)}(p_1'-p_1+Q)\delta_{R_2}^{(4)}(p_2'-p_2-Q)}{Q^2} d^4Q + \cdots ,$$

with $\delta_a^{(1)}(k)=(1/2\pi)\int_{-a/2}^{+a/2}\exp(ikx)dx=\sin(ka/2)/\pi k$ and the size $a=diam(R)$ [both regions have been assumed to be of equal (directional) size]. Only in the limit $a\to\infty$ becomes $\delta_a\to\delta$ and the energy-momentum is exactly conserved at the vertex. On close inspection we note that there is no reason why for arbitrary time resolution Δt the energy change of the emittor or absorber line $\approx\Delta Q_0$ should be within the uncertainty limits imposed by quantum theory (that is, $\Delta t\Delta Q_0 \ll \hbar$), as was argued by Fierz and Källen; for any Δt there exist ΔQ_0's such that $\delta_{\Delta t}(p_{1,0}'-p_{1,0}+Q_0)\neq 0$ for $\Delta t\Delta Q_0>1$. Therefore, the above argument via phase-space considerations remains ambiguous. Arguments for causality violation in quantum mechanics have indeed been put forward by Hegerfeld[5] and Rubin.[6]

In what follows it will be shown that, despite nonlocal contributions to amplitudes, causality is not violated in quantum-field theory. This will be done by explicitly calculating the propagation of a light pulse. It can be shown that the broadening of extremely short pulses (of a few wavelengths, corresponding to 10–100 fs in duration) is only due to preparation and measurement uncertainties and cannot be derived from any nondispersive character of the vacuum of quantum-field theory.

The propagation of a light pulse traveling in one direction (say, along the x_1 axis) can be approximated as a $(1+1)$-dimensional problem by neglecting the packet spread in the directions perpendicular to the direction of propagation. The corresponding initial value problem for the wave equation in $(1+1)$ dimensions will be solved by Green's-function techniques.

The Feynman propagator (causal Green's function) stemming from the Wick decomposition of field amplitudes for a (scalar) photon in $(3+1)$ dimensions is given by

$$\begin{aligned}\Delta_c^{(3+1)}(x) &= \frac{1}{2\pi}\delta_+(-x^2) = -\frac{i}{(2\pi)^2}\frac{1}{x^2-i\epsilon} \\ &= \frac{1}{2}[\Delta_R(x)+\Delta_A(x)+\Delta^1(x)] = \frac{1}{4\pi}\left[\delta(x^2)-\frac{i}{\pi}\mathcal{P}\frac{1}{x^2}\right] \\ &= \frac{1}{4\pi}\left[\frac{1}{2R}[\delta(t-R)+\delta(t+R)]-\frac{i}{\pi}\mathcal{P}\frac{1}{t^2-R^2}\right],\end{aligned}$$

where $\mathcal{P}$ stands for the principal value and $x^2=t^2-R^2$ with $R^2=x_1^2+x_2^2+x_3^2$ (see, for instance, Bogoliubov and Shirkov[7] for details). Upon integration of the coordinates one obtains the causal Green's functions in $(2+1)$ dimensions

$$\Delta_c^{(2+1)}(x)=\frac{1}{4\pi}\left[\frac{\theta(|t|-P)}{|t^2-P^2|^{1/2}}-i\mathcal{P}\frac{1}{|t^2-P^2|^{1/2}}\right],$$

where $P^2=x_1^2+x_2^2$, and the causal Green's function in $(1+1)$ dimensions (from now on, the spacial index is dropped, such that $x\equiv x_1$)

$$\Delta_c^{(1+1)}(x,t)=\frac{1}{4}\left[\theta(|t|-|x|)-\frac{i}{\pi}\mathcal{P}\ln|x^2-t^2|\right].$$

The initial value problem can be defined as follows: (i) propagation in the vacuum with no sources present (corresponding to the free-wave equation), (ii) the (scalar) photon is represented by a square-well pulse of width σ and zero velocity at $t_0=0$ (this configuration represents two pulses transversing each other from opposite directions and with opposite velocities)

$$u(x,t_0=0)=\theta\left[\frac{\sigma}{2}-|x|\right],\quad \frac{\partial u}{\partial t}(x,t_0=0)=0 .$$

The shape of the light pulse at a later time t can either be found by insertion of the explicit form of the disper-

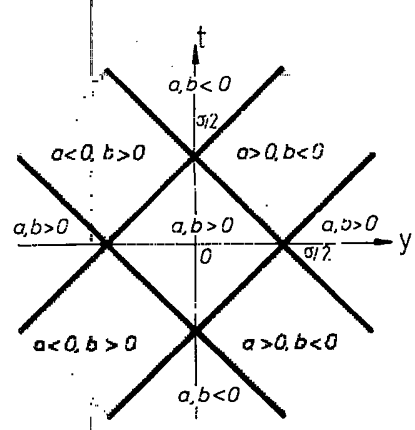

FIG. 1. Schematic representation of the domains of the argument of $\ln(a/b)$ with $a=y^2-[(\sigma/2)-t]^2$ and $b=y^2-[(\sigma/2)+t]^2$. Shaded areas indicate that $\mathrm{Im}\,\ln(a/b)\neq 0$.

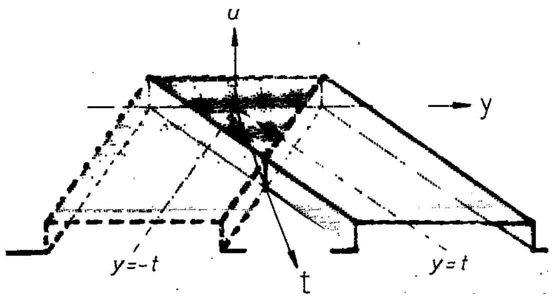

FIG. 2. Dispersion-free propagation of a square-well light pulse.

sion $\omega(k)$ or in configuration space by integration over the initial values[8]

$$u(y,t)=2\,\mathrm{Re}\int_{-\infty}^{+\infty} \left.\frac{\partial\Delta_c^{(1+1)}(x-y,\,t_0-t)}{\partial t}\right|_{t_0=0} \times\theta\left[\frac{\sigma}{2}-|x|\right]dx\ ,$$

where Re denotes the real part of the integral and

$$\frac{\partial\Delta_c^{(1+1)}(x-y,-t)}{\partial t} =\frac{1}{4}\left[\delta(|t|-|x-y|)-\frac{i}{\pi}\mathcal{P}\frac{2t}{(x-y)^2-t^2}\right].$$

Upon integration the solution $u(y,t)$ is given by

$$u(y,t)=\frac{1}{2}\left\{\theta\left[\frac{\sigma}{2}-|y+t|\right]+\theta\left[\frac{\sigma}{2}-|y-t|\right] -\mathrm{Re}\left[\frac{i}{\pi}\ln\left[\frac{y^2-(\sigma/2-t)^2}{y^2-(\sigma/2+t)^2}\right]\right]\right\}$$

Notice that the last term on the right-hand side only contributes if the argument of the integral is negative, yielding a factor of $\ln(-|a|)=i\pi+\ln|a|$. For the above argument this is only the case for the four regions indicated in Fig. 1. The final solution can be written down as

$$u(y,t)=\begin{cases}\theta((\sigma/2)-|y+t|)\\ \theta((\sigma/2)-|y-t|)\end{cases}.$$

It is drawn in Fig. 2.

A similar problem is the calculation of the field $v(y,t)$ from a very short-lived line charge $q(x,\tau)=(1/2\pi)\delta(\tau)\theta((\sigma/2)-|x|)$. Integration over the inhomogeneous wave equation yields

$$v(y,t)=8\pi\,\mathrm{Re}\int_0^t d\tau\int_{-\infty}^{+\infty}dx\ \Delta_c^{(1+1)}(x-y,\,t-\tau)q(x,\tau) =\theta\left[\frac{\sigma}{2}+|t|-|y|\right].$$

Again, local causality for the propagation of a quantized field is confirmed.

The main result of the above calculation is that the width of a wave packet traveling in the vacuum remains constant (contrary to a spreading of the packet if acausal effects were to be expected). The packet does not decay in time. Hence the vacuum of quantum-field theory remains dispersion-free for the propagation of light and no acausality occurs. It should be worth investigation under which circumstances an experimental test validates these predictions.

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[1]The notions of causality and locality (or separability) are used in a different context: in an EPR-type configuration with a spin singlet state decaying into two spin-$\frac{1}{2}$ states, let $(\mathbf{s}_i\cdot\mathbf{a})(x)$ denote a measurement of the spin $\mathbf{s}$ of the ith subsystem in the direction $\mathbf{a}$ at the position x, then quantum mechanics predicts an expectation value

$$\langle(\mathbf{s}_1\cdot\mathbf{a})(0)(\mathbf{s}_2\cdot\mathbf{b})(x)\rangle=-\mathbf{a}\cdot\mathbf{b}\ ,$$

independent of the separation x^2 of the two subsystems. This and other consequences of quantum nonseparability (among them the most prominent "collapse of the wave packet") has been the basis of speculations concerning a "nonlocality of quantum mechanics."

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