Impossibility of measuring faster-than-c signaling by the Scharnhorst effect

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It is argued that the Scharnhorst effect cannot in principle give rise to measured signal velocities larger than that of light in vacuum.

Scharnhorst [1] and subsequently Barton [2] (see also ref. [3]) have shown that the refractive index of the vacuum between parallel plates can be less than unity for all frequencies $\omega < mc^2/\hbar$, where m is the electron mass [#1]. This arises from the decreased polarizability of the vacuum between the plates via virtual electron–positron pair production, suggesting the possibility in principle of faster-than-c signal propagation.

Along these lines one could also conceive of a decreased polarizability in the presence of nuclei with charges above the critical charge ($Z \geqslant Z_{cr} \approx 164$–$172$), associated with the charged vacuum as ground state [7]. In the vicinity of these nuclei, this would yield a shift of the refractive index given by $\Delta n(Z) = [1-\alpha\Pi(Z)]^{1/2} - [1-\alpha\Pi(Z=0)]^{1/2} \approx -\frac{1}{2}\alpha\Delta\Pi$ ("dived states"), where α and Π are the fine structure constant and vacuum polarization, respectively, yielding a contribution to $\Delta n \approx \Delta c/c = -O(\alpha)$ which is one order in the fine structure constant *lower* than the Scharnhorst effect.

The decrease of n can be related to the negative expectation value of the electromagnetic energy density u between the plates via the effective potential method. In terms of the effective potential $\Gamma(a_\mu)$ (A_μ stands for the expectation value of the electromagnetic field) and for vanishing momenta, u may be written as [8] $\Gamma(A_\mu) = -(2\pi)^4\delta(0)u(A_\mu)$. It can be shown that u is given by the sum containing all one-particle irreducible (1PI) Green's functions with vanishing external momenta:

$$u(A_\mu) = -\sum_{n=2}^{\infty} \frac{1}{n!}\tilde{\Gamma}^{(n)}(0, \ldots, 0)A_{\mu_1} \ldots A_{\mu_n} .$$

The one-loop four-point Green's function has been used by Scharnhorst to calculate the photon polarizability via electron–positron pair production. (Of course, this does not rule out cancellations of terms $\tilde{\Gamma}^{(n)}A_{\mu_1} \ldots A_{\mu_n}$ *within* the summation.)

The fact that $u<0$ is associated with an attractive force between the plates (Casimir effect), and has already been invoked in the somewhat similar context of wormholes and time machines [9]. The Scharnhorst effect is much too small to imagine experimental tests, but it nevertheless raises an important point of principle about whether it is possible for signal velocities to exceed the velocity c of light in vacuum [1–3]. In this letter we show that the Scharnhorst effect does not in principle allow one to measure signal velocities $v > c$.

A direct measurement of v would involve a measurement of the time t required for propagation over a fixed distance L': $v = L'/t$. Due to any uncertainty Δt in the measured time, the value of v deduced in

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[#1] Note, however, that earlier claims of propagation velocities larger than c due to polarizability via virtual electron–positron pair production in the presence of strong but slowly varying ($\lambda \gg \lambda_c = \hbar/mc$) external fields have been proven to be incorrect in ref. [4]. A thorough discussion of the dielectric function and related questions of causality and stability has been given in ref. [5]. The distinction between signal and group velocity (as well as other notions of velocity) is a longstanding concern and early work was reviewed in ref. [6].

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this way is uncertain by the amount $\Delta v = L'\Delta t/t^2 = c^2\Delta t/L'$. For the present discussion we take $L' = L$, the plate separation in the Casimir vacuum considered by Scharnhorst and Barton; this *minimizes* the uncertainty in the measured velocity [#2].

The uncertainty Δt will be limited, aside from practical considerations, by the uncertainty in time it takes to switch on a signal. For instance, if a photon emitter is switched "on" at time $t=0$, by exciting an atom, there is a fundamental uncertainty as to when the quantum jump and photon emission will occur. In the most optimistic case imaginable $\Delta t = 1/\omega$, where ω is the frequency of the signal. We could also choose Δt to be the radiative lifetime of an atom, but this would lead to a larger value of Δv, and for the present argument we wish to examine the situation most favorable to the possibility of faster-than-c signaling. Thus we take $\Delta v = c^2/\omega L = c\lambda/L$, where λ is the wavelength of the signal radiation.

The change in c predicted by the Scharnhorst effect is $\Delta c = \kappa\alpha^2\hbar^4/m^4c^3L^4$, where α is the fine structure constant ($\cong 1/137$) and $\kappa \cong 1.3\times10^{-2}$ [2]. Thus the ratio of the uncertainty in a measured value of v to Δc is

$$\frac{\Delta v}{\Delta c} = \frac{1}{\kappa\alpha^2}\frac{\lambda}{L}\left(\frac{mcL}{\hbar}\right)^4 = \frac{\lambda}{\kappa\alpha^2}\frac{1}{\lambda_c}\left(\frac{L}{\lambda_c}\right)^3, \qquad (1)$$

where $\lambda_c = \hbar/mc$ ($\cong 3.9\times10^{-11}$ cm) is the electron Compton wavelength. Barton has noted that the assumption $\lambda/L \ll 1$ is required not only in order to permit clean thought experiments, but also to justify the assumption of a local refractive index. The theory also assumes $\lambda \gg \lambda_c$; in order again to be as optimistic as possible in our analysis of the Scharnhorst effect, and to minimize Δv, we use $\lambda = \lambda_c$ in (1):

$$\frac{\Delta v}{\Delta c} \geqslant \frac{1}{\kappa\alpha^2}\left(\frac{L}{\lambda_c}\right)^3 \approx 1.5\times10^6\left(\frac{L}{\lambda_c}\right)^3. \qquad (2)$$

Morris et al., in their discussion of wormholes and time machines [9], have noted that plate separations less than the Compton wavelength "might well be forbidden". In this connection note that if $L=\lambda_c$, then $t=\hbar/mc^2 \equiv 1/\omega_{max} < 1/\omega = \Delta t$. (Recall that the Scharnhorst theory assumes $\omega < \omega_{max}$.) In other words, if $L = \hbar/mc$ the uncertainty in the time it takes a signal to traverse the distance L is larger than $t = L/c$. This would imply $\Delta v > c$. A more reasonable lower limit on L might be the Bohr radius $a_0 = \lambda_c/\alpha$, in which case

$$\frac{\Delta v}{\Delta c} \approx \frac{1}{\kappa\alpha^5} \approx 13.7\times10^{12}. \qquad (3)$$

At such tiny separations the distinction between the plates as macroscopic objects becomes blurred, of course, and repulsive forces associated with overlapping electron wavefunctions come into play to increase u and weaken and remove the Scharnhorst effect. In any case it is clear that the uncertainty in the measured propagation velocity will always be enormously larger than the correction to c associated with the Scharnhorst effect.

We conclude, therefore, that no measurement of the faster-than-c velocity of light predicted by the Scharnhorst effect is possible. It is worth noting that our conclusion assumes the small value of the fine structure constant determined by e, $\hbar$ and c. In a universe in which α were large, our conclusion would not hold. In other words, our conclusion rests on the small value of the fine structure constant rather than the basic dynamical laws of physics. This is not the first example where a violation of causality is ruled out by the values of constants rather than dynamical laws [10].

[#2] L' could in principle be made arbitrary large by allowing a light pulse to bounce back and forth repeatedly between the plates. This could, however, not be used for faster-than-c signaling.

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