Comments

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Comment on "Comment on 'Quantum cosmology and the initial state of the universe' "

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The algorithmic complexity approach to cosmology is discussed. Emphasis is given to the time evolution of the complexity of the Universe. In certain limits, the complexity increases and is not bounded from above, as was implicitly suggested by earlier research. There is no need of "arbitrary choices" in the later stages of the Universe in order to explain a great number of independent physical parameters. Even if, for instance, by some finitistic condition, one could impose conservation of complexity, this measure is intractable and thus hardly useful for an operationalization.

In a recently published Comment[1] on the wave function of the Universe,[2] Woo raises the following interesting question: Since macroscopic physics shows such a rich phenomenology, how could all this derive from a low-complex evolution?[3] (This question somewhat resembles the problem of symmetry breaking in gauge-invariant field theories.) He approaches this question with relatively new and powerful techniques from algorithmic complexity theory.[4–6] Roughly speaking, algorithmic complexity is the minimal program length necessary to represent an entity, such as an equation of motion, initial configurations, or boundary conditions. Woo assumes that the following relation holds: (algorithmic complexity)$\approx$(maximal number of independent macroscopic observables). He then argues that if the combined complexity measure of the evolution equations and the initial values is of the order of 10^5 bits, this could account only for so many independent physical parameters. If, on the other hand, one could prove that there exist more parameters of that type, these would have to come from what he calls "arbitrary choices" in some later stage of the Universe. One might speculate that the source for "arbitrary choices" is a kind of "quantum oracle" which occurs with every "random" (I would rather prefer the term "undecidable") microscopic event which contributes at least one bit of information. In this vision, the diversity of the macrocosmos emerges from, and is constantly created by, microphysical indeterminism.

Despite my principal support of Woo's complexity-based approach to cosmology, I would like to critically comment on his considerations as follows.

(i) Woo's implicit assumption is that algorithmic complexity without what he calls "arbitrary choices" is conserved throughout the evolution of the Universe, or at least does not increase with time. This assumption is highly nontrivial. One may derive some evidence for it from a statement cited in Woo's Comment, that "a theory cannot predict more than it contains," which is a popularized translation of a quantitative reinterpretation[7] of Gödel's celebrated incompleteness theorem.[8] However, as it stands, this statement is incorrect in important limits.

For example, for discrete times t_i, $i=1,2,3,\ldots$ and on universal computational devices it is possible to produce a random real with an infinite algorithmic complexity from a computable function and computable input with a finite combined initial complexity in the limit $t_i \rightarrow \infty$. Indeed, Chaitin[4] has published a LISP program which (given a universal computer) would do just that.

In quantum theory and gravity the time evolution is continuous and such limits resulting in a sort of "creation of complexity" could occur even for finite times $t<\infty$.

Furthermore, one could utilize recent results[9] of recursive analysis, roughly stating that Hermitian time evolutions from computable initial values preserve computability (and thus the finiteness of algorithmic complexity) if the corresponding space is bounded. Otherwise, the system description may again become uncomputable.

All these considerations boil down to the fact that by infinite means it is possible to "generate" algorithmic complexity. The same does not hold for finite means.[10]

These considerations with respect to the "generation of algorithmic complexity" facilitate speculations concerning the growing complexity of living organisms (corresponding to more complex DNA's) throughout the evolution of species,[11] or the increase of entropy measures.[12]

(ii) Even if it were in principle possible to impose limits on the algorithmic complexity of the wave function of the

Universe (which, as has already been pointed out, is conceivable only with finitistic assumptions), such limits would in general be of not much practical help. This is due to the fact that complexity measures are in general intractable and thus uncomputable. In other words, Woo's argument is not constructive in the sense that in general there is no systematic way to derive laws and parameters from a particular phenomenology. Since physical statements which are not constructive are not subject to any test, this is rather unsatisfactory, to say the least.

One could, of course, always introduce heuristic complexity measures[12] (such as the ones derived from standard compression algorithms), but these can never give the certainty which would be needed for a definitive decision on a finitistic organization of our Universe.

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[1]C. H. Woo, Phys. Rev. D **39**, 3174 (1989).

[2]J. B. Hartle and S. W. Hawking, Phys. Rev. D **28**, 2960 (1983); A. Vilenkin, *ibid.* **37**, 888 (1988); T. Vachaspati and A. Vilenkin, *ibid.* **37**, 898 (1988).

[3]However, as is well documented for the Mandelbrot set, very simple computable functions may have very complex representations.

[4]G. J. Chaitin, *Algorithmic Information Theory* (Cambridge University Press, Cambridge, England, 1987).

[5]A. K. Zvonkin and L. A. Levin, Russ. Math. Sur. **25**, 83 (1970).

[6]V. M. Alekseev and M. V. Yakobson, Phys. Rep. **75**, 287 (1981); see also G. Kreisel, Synthese **29**, 11 (1974); R. P. Feynman, Int. J. Theor. Phys. **21**, 467 (1982); C. H. Woo, Phys. Lett. **168B**, 376 (1986); M. Minsky, Int. J. Theor. Phys. **21**, 537 (1982).

[7]Assume x stands for a theorem and $H(x)$ for its algorithmic complexity. Then for any finitely axiomatized formal theory T (assumed to be sound), there is a constant $k > H(T) + O(1)$ such that no statement of the form $H(x) > k$ is provable in T. For details see Ref. 4.

[8]Gödel's incompleteness theorem defines a universal recursively enumerable (and by Church's hypothesis) computable set S of numbers and states that for every finitely axiomatized formal theory T (assumed to be sound) there is a number n for which the fact $n \notin S$ is true but not provable in T.

[9]M. B. Pour-El and I. Richards, Adv. Math. **39**, 215 (1981); **48**, 44 (1983); **63**, 1 (1987).

[10]R. O. Gandy, *Limitation to Mathematical Knowledge*, proceedings of the Logic Colloquium '82, edited by D. van Dalen, D. Lascar, and J. Smiley (North-Holland, Amsterdam, 1982).

[11]G. Chaitin, Sci. Am. **259**, 52 (1988).

[12]K. Svozil, Phys. Lett. A **140**, 5 (1989).