IL NUOVO CIMENTO VOL. 96 B, N. 2 11 Dicembre 1986

Operational Concepts and the Transformation Laws of Space-Time Co-ordinates in Arbitrary Dispersive Media.

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(ricevuto il 26 Febbraio 1986)

Summary. — The concept of operational parameters is discussed. Operational parameters are compared with extrinsic or hidden variables. It is shown that, by applying Einstein's synchronization conventions, the transformation properties of operational space and time parameters depend on the dispersion (energy momentum) relation of the system. In particular, with nonlinear forces acting in a medium, transformation laws need not necessarily be of the Lorentz form.

PACS. 04.20. – General relativity.

1. – Introduction.

Among other aspects, there are two principal ways of describing a system: the *operational*- (or *intrinsic*-) parameter description and the extrinsic-parameter description. Heuristically speaking, the operational-parameter description may include every parameter which can actually be measured *from within the system* in the sense that devices can be constructed and processes can be imagined which would lead to a measurement of such a system parameter. This would eliminate idealistic concepts or mere hopes that it would eventually be possible to find the « true and real » value of a system parameter with an associated uncertainty zero. Such illusionary ways of description are referred to as *extrinsic*, since they would require a point outside the world, from which it is possible to look at the world, without disturbing it at all. Its parameters may be *hidden parameters* from within the system. Since early Greek philosophers such as PLATO and ARCHIMEDES (1) recorded such considerations, it is quite obvious that natural sciences have to rely exclusively on operational-parameter descriptions. More recently, the use of operational parameters has been stressed for statistical physics (2-7). However, to my knowledge, no comprehensive study on these notions has been undertaken yet. The operational-parameter description is the only physically meaningful representation of a system and it has many advantages. One of these advantages is its pragmatic approach: in particular, the observer (the measurement apparatus) need not and cannot be excluded from the observed system. Moreover, the operational-parameter description needs no knowledge about the closedness of a system to its environment, since it does not attempt a comprehensive, inclusive description of that particular system. This ignorance is indeed inherent for any parameter description, since one never knows for sure the forces acting in a system, nor will it ever be possible to project future developments of knowledge, which may eventually yield corrections within the range of present uncertainty. There seems to be an unavoidable consequence: the acceptance of the preliminarity of all scientific findings forces investigations to consider all possible operational descriptions and their underlying structure. There cannot be anything like a preferred parameter description for other reasons than subjective ones.

In sect. **2** a very brief introduction to the operational-parameter description is given. It is shown how a sort of *quasi-extrinsic*-parameter description can be developed. In sect. **3** these concepts are applied to space-time parameters in the field-theoretic description of quantized media. It is shown that their transformation properties depend on synchronization conventions and on the dispersion (energy-momentum) relation in a particular medium. In sect. **4** speculations about a Lorentz-invariant vacuum medium and experiments for its detection are introduced.

2. – The operational concept.

For clarity and consistency, the operational parameter description is introduced. Assume a system S. Then an operational parameter description $P(S) = \{S; p_1, ..., p_n, ...\}$ with parameters p_i (it is assumed that the set $P(S)$

(1) B. RUSSEL: *A History of Western Philosophy* (Allen and Unwin, London, 1964).
(2) B. MISRA: *Proc. Natl. Acad. Soc. USA*, **75**, 1627 (1978).
(3) B. MISRA, I. PRIGOGINE and M. COURBAGE: *Physica (Utrecht) A*, **98**, 1 (1979).
(4) I. PRIGOGINE: *Science*, **210**, 777 (1984).
(5) C. M. LOCKHART: Ph.D. Thesis, University of Texas at Austin (1981).
(6) C. LOCKHART, B. MISRA and I. PRIGOGINE: *Phys. Rev. D*, **25**, 921 (1982).
(7) H. NARNHOFER, M. REQUART and W. THIRRING: *Commun. Math. Phys.*, **92**, 247 (1983).

is countable, but this is not necessary) contains parameters which could at least in principle be measured by devices and processes available in that system. According to EINSTEIN [8] it can be said that each element of $P(S)$ corresponds to an *element of reality*.

Such a parameter description may be consistent and unambiguous (but it need not be) if both the system and the devices and processes are consistent and unambiguous. Since this is not a necessary condition, operational descriptions are quite arbitrary and have to be selected by economic, symmetric and progressive reasons (this introduces a subjective element to a selection procedure). Here *progressive* means that the description is able to foster future developments in that area, as has been intensively discussed by LAKATOS [9]. It is only a belief that the most economic and symmetric parameter description is the most progressive one. As historic examples show (for instance the Ptolemean *vs.* the Kopernican system), this need not always be the case.

Next I shall try to construct examples of operational-parameter descriptions and introduce the concept of *quasi-extrinsic* (almost extrinsic) descriptions. Let us consider two systems S_1 and S_2, an interaction I acting in both of them, and two associated operational-parameter descriptions $P(S_1, I)$ and $P(S_2, I)$. We shall define the system S_2 *approximately closed* with respect to S_1 and to interaction I, if S_2 responds only «slightly» to changes in S_1. Formally, this situation can be characterized by

$$\frac{\delta P(S_2, I)}{\delta p_i} \simeq 0 \qquad \forall p_i \in P(S_1, I)\,. \tag{2.1}$$

Using the language of cybernetics, this is identical to saying that a system S_2 is approximately autonomous with respect to S_1 if the effect of its output affects its input only slightly, such that no feedback loop via S_1 occurs [10].

A parameter description is called *independent*, when all parameters commute with each other with respect to the Poisson bracket:

$$[p_i, p_j] = 0 \qquad \text{with } p_i, p_j \in P(S, I)\,. \tag{2.2}$$

When the system S_1 is approximately closed with respect to S_2 and *vice versa*, the corresponding operational-parameter descriptions in both systems will be (almost) independent. Otherwise, they would not be measurable simultaneously.

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(8) A. EINSTEIN, B. PODOLSKY and N. ROSEN: *Phys. Rev.*, **47**, 777 (1953).

(9) I. LAKATOS: *The Methology of Scientific Research Programs, Philadelphia Papers*, Vol. 1, edited by J. WORRALL and G. CURRIE (Cambridge University Press, Cambridge, 1978).

(10) F. VARELA: *The Principles of Biological Autonomy* (North-Holland, New York, N. Y., 1980).

A parameter description is called *complete*, if no parameters p exist such that p is independent with respect to $P(S, I)$, but does not belong to it:

$$\neg \exists p : [p, P(S, I)] = 0 \wedge p \notin P(S, I) . \tag{2.3}$$

Generally, if the parameter descriptions $P(S)$ and $P(S, I)$ are complete and independent,

$$P(S) \supset P(S, I) , \tag{2.4}$$

since there may be more interactions than I. Clearly, what physics always wants (and sometimes claims) is just an independent and complete parameter description $P(W)$ of the whole world W with all interactions taken into account (assuming an arbitrary but finite subset E of W, the cosmological principle can, for instance, be formulated as $P(W) \supset P(E)$).

Assume two systems S_1 and S_2 and two interactions I_1 and I_2, acting in both of them. Assume further that S_1 and S_2 are approximately closed with respect to one interaction, say I_1:

$$\frac{\delta P(S_1, I_1)}{\delta P(S_2, I_1)} \simeq \frac{\delta P(S_2, I_1)}{\delta P(S_1, I_1)} \simeq 0 . \tag{2.5}$$

We shall spoil the symmetry now by requiring that one system, say S_1, is sensitive to interactions I_2 in S_2, whereas S_2 is not:

$$\frac{\delta P(S_1, I_2)}{\delta P(S_2, I_2)} \not\simeq 0 \wedge \frac{\delta P(S_2, I_2)}{\delta P(S_1, I_2)} \simeq 0 . \tag{2.6}$$

Hence, effectively one almost closed system S_2 (with respect to S_1) and one open system S_1 is obtained. The latter system S_1 is a close realization of Archimedes' point with the system S_2 and the interaction I_1 to be described (here, I_2 serves as a reference interaction). Since observations in S_1 will not affect S_2 too much, an operational-parameter description $P(S_2, I_2)$ will be called *quasi-extrinsic*. Parameters in $P(S_2, I_2)$, which cannot be measured by I_1 in S_2, are called *hidden* parameters in $P(S_2, I_1)$.

In this context, the extrinsic-parameter description might be defined via a limit: a parameter description $P(S_1, I_2)$ from S_2 is called *extrinsic*, if S_1 and S_2 are totally closed with respect to both interactions I_1 and I_2. Clearly, this is impossible to realize, since there cannot be any exchange between systems without altering the states of both.

In what follows I shall give an example of a quasi-extrinsic configuration: assume a pool filled with water, which will serve as system S_2. Let us assume further an optical instrument recording electromagnetic radiation as part of our system S_1, and the interactions I_1 and I_2, being identified with water wave interaction (which only acts in the pool S_2) and electromagnetic interaction, respectively. Since light does not affect water wave dynamics appreciably, but changes the state of the optical instrument, a realization of the described cybernetic model is obtained, with the optical instrument yielding a quasi-extrinsic view of the pool.

Now suppose there is a waver-flea (Cladocera) living on the surface of the water. Suppose further that this creature is blind, *i.e.* it is not able to employ electromagnetic radiation. Then its operational parameter description will almost certainly purely have to rely on water wave dynamics as operational device. On the other hand, another creature (such as a bird), seeing the surface of the water by light, will be able to choose between the water wave dynamical description by electromagnetic scanning (*i.e.* photography or that like) and wave scattering; both are operational for this creature. Whereas the water-flea, if it is not imaginative enough, will always wonder about the form of its world and its embedding and limitations, the air creature will immediately overlook the situation, thereby rather using its eyes than producing wave scattering. That of course does not mean that the water-flea, in principle, can never detect light; it is just that electromagnetic effects yield such small contributions to phenomenology that they are difficult to detect.

For example, the most obvious and evident operational-parameter description with water waves is by a sort of quantization with the help of Poisson's brackets ([11])

$$[x, p] = k \tag{2.7}$$

for all canonical conjugates x and p. Here k stands for Planck's constant in the water medium, which is the smallest action possible by water waves. For water ([12]), $k =$ (kinetic energy per atom)/(wave frequency) $\approx 6 \cdot 10^{-22}$ erg s is much greater than Planck's constant in vacuum, h. The quotient $|h/k| \approx 10^{-5}$ shows that the electromagnetic resolution of canonical conjugate parameter in vacuum is five orders of magnitude higher than for water waves.

The question arises what is the use of such a water wave description? One answer is certainly pedagogical; we know it better: from our quasi-extrinsic point of view, water is just made out of molecules forming waves. We stick to what appears most evident to us—but not so for the water flea! Thus probably we can learn from this example that we have to be very careful by using parameter descriptions and relying completely on their conclusiveness and uniqueness.

As an example I shall try to answer the question which extrinsic-parameter description might be used for space, time and energy in a quantized medium.

([11]) H. Goldstein: *Classical Mechanics* (Addison-Wesley, Reading, Mass., 1959).
([12]) F. S. Crawford: *Berkeley Physics Course*, Vol. **3** (Waves) (McGraw-Hill, New York, N. Y., 1965), Sect. **6**'2 and **6**'3.

3. – The setting of intrinsic scales for space, time and energy.

So far nothing has been said about the formal structure of theories corresponding to parameter descriptions P. As a postulate it is assumed that any theoretical structure can be characterized ([13]) by an algorithm A, encoded by a string formed by some alphabet a. It could be expected that there is a one-to-one (bijective) correspondence $P \equiv A$. In this case we may say that A is *inclusive* with respect to P. However, this saying is probably of not much use, since one never knows if P is an inclusive representation of the empirical data of reality (or if one does not accept realism in this context: one never knows if to a later time there will not be a more inclusive P' of the same system, such that $P(S) \subset P'(S)$). The only qualitative correspondence between P and A seems to be a historic one: to every historic moment, a temporal correspondence of some representation of the world P with some explanation of the world A is attempted.

Whether algorithms can be accepted which do not contain all empirical content, such that $P \supset A$, or whether they contain hidden variables such that $P \subset A$ or a mixture of both cases for different elements of reality in the sense of Einstein ([9]) is quite arbitrary. If these algorithms are reducible (such that a short cut is possible to obtain the parameter description), or NP-complete (such that they can be solved with an arbitrary number of parallel computational paths and in polynomial time but are computational irreducible), or PSPAC-complete (such that they can be solved with polynomial storage capacity, but require exponential time), or else, are still open questions which have been edged on only recently ([14]). I shall not pursue them here.

As an example I shall rather discuss a theory of a quantum medium which may yield the well-known covariant transformation laws of space-time and energy in some *operational* parameter description, but is certainly not manifestly covariant from an *extrinsic* or *quasi-extrinsic* point of view. It can be seen that certain symmetries such as general covariance emerging in specific parameter descriptions may not be evident for others.

For further considerations it is assumed that it is, at least in principle, possible to construct measurement apparata for certain observables and parameters such as space, time or internal quantum numbers out of elementary processes in a system. In that way, frames of reference can be defined in an operational way.

A) Scales within a system. In what follows a time scale will be defined operationally. A model atomic clock is constructed which is at rest with respect

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([13]) J. E. HOPCROFT and J. D. ULLMAN: *Introduction to Automata Theory, Languages, and Computation* (Addison-Wesley, Reading, Mass., 1979).
([14]) ST. WOLFRAM: *Phys. Rev. Lett.*, **54**, 735 (1985).

to the rest of the system. Times t_A and t_B are assigned to its initial and final states $|\psi t_A\rangle$ and $|\psi t_B\rangle$. Both t_A and t_B may be arbitrary real numbers with $t_A < t_B$. It is assumed that the state of the clock can be expanded into a sum of orthonormal state vectors $|\mathrm{I}\rangle$ and $|\mathrm{II}\rangle$:

$$|\psi t\rangle = |\mathrm{I}\rangle c_{\mathrm{I}}(t) + |\mathrm{II}\rangle c_{\mathrm{II}}(t)\,, \tag{3.1}$$

where $c_{\mathrm{I}}(t) = \langle \mathrm{I}|\psi t\rangle$ and $c_{\mathrm{II}}(t) = \langle \mathrm{II}|\psi t\rangle$, respectively. It is further assumed that the dynamics of the system is given by

$$i\,\frac{\mathrm{d}}{\mathrm{d}t}\,c_{\mathrm{I}}(t) = \varepsilon c_{\mathrm{I}}(t) - \omega c_{\mathrm{II}}(t)\,, \tag{3.2a}$$

$$i\,\frac{\mathrm{d}}{\mathrm{d}t}\,c_{\mathrm{II}}(t) = -\,\omega c_{\mathrm{I}}(t) + \varepsilon c_{\mathrm{II}}(t)\,. \tag{3.2b}$$

Since $|\mathrm{I}\rangle$ and $|\mathrm{II}\rangle$ are no eigenstates of the Hamiltonian, an oscillation between them occurs if the clock was initially at the time t_A in the state $|\mathrm{I}\rangle$. After a short calculation the probability of finding this clock in the same state $|\mathrm{I}\rangle$ at a later time t_B is found to be

$$|c_{\mathrm{I}}(t_B)|^2 = |\langle \mathrm{I}t_B|\mathrm{I}t_A\rangle|^2 = \cos^2[\omega(t_B - t_A)]\,. \tag{3.3}$$

Since the time span $t_A - t_B$ has been defined arbitrarily, the frequency parameter ω in eq. (3.2) has to be adjusted so that the measurable transition probability (3.3) is satisfied. By identifying ω with the frequency $\omega(p = 0)$ of the dispersion relation of the clock for zero momentum p, we have fixed the energy scale in that particular system. Having thus fixed time and energy scales in one point of the system, other clocks at rest with respect to that system can be synchronized by Einstein's conventions ([15,16]). Similarly, the definition of a length scale is performed by carrying a reference rod around and comparing all other length scales with this standard.

B) Comparing intrinsic scales. Having defined intrinsic scales for clocks at rest with respect to a given system or frame of reference, the attention is drawn to the relation of scales in different frames of reference. Again an operational approach is chosen by considering two clocks, one at rest with respect to the frame S_1 and the other with respect to a second frame S_2. In order to compare scales, a transition of the state of the clocks that can be measured from both systems has to be considered: assume a single clock, say the clock at rest in S_2, described as above by the amplitudes $\langle \psi_1 t_{1B}|\psi_1 t_{1A}\rangle$ in S_1 and

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([15]) A. EINSTEIN: *Ann. Phys.* (*N. Y.*), **17**, 891 (1905).
([16]) R. MANSOURI and R. U. SEXL: *Gen. Rel. Grav.*, **8**, 497 (1977).

$\langle \psi_2 t_{2B} | \psi_2 t_{2B} \rangle$ in S_2. As the square of these amplitudes has a probabilistic interpretation and by identifying $|\mathrm{I}_2\rangle = |\mathrm{I}_1\rangle = |\mathrm{I}\rangle$ and $|\mathrm{II}_2\rangle = |\mathrm{II}_1\rangle = |\mathrm{II}\rangle$, the following identities are obtained:

$$|\langle \mathrm{I} t_{2B} | \mathrm{I} t_{2A} \rangle|^2 = |\langle \mathrm{I} t_{1B} | \mathrm{I} t_{1A} \rangle|^2 , \tag{3.4a}$$

$$|\langle \mathrm{II} t_{2B} | \mathrm{II} t_{2A} \rangle|^2 = |\langle \mathrm{II} t_{1B} | \mathrm{II}_{1A} \rangle|^2 . \tag{3.4b}$$

These relations hold true for all times and the clock can be described by the same type of evolution equation from both frames. With the initial condition $c_{1\mathrm{I}}(t_{1A}) = c_{2\mathrm{I}}(t_{2A}) = 1$, the time development of the $|\mathrm{I}\rangle$-state is given by

$$\cos^2 [\omega_1 (t_{1B} - t_{1A})] = \cos^2 [\omega_2 (t_{2B} - t_{2A})] . \tag{3.5}$$

Here, the indices 1 and 2 always indicate « measured in S_1 and S_2 ». By comparing the arguments of the cosine and taking the limit of infinitesimal time differences, the time dilatation is obtained:

$$\frac{\mathrm{d}t_2}{\mathrm{d}t_1} = \frac{\omega_1}{\omega_2} . \tag{3.6}$$

As in the case of time scales, there is an arbitrariness in the transformation of space scales, since the procedures for comparing two scales in two different frames are not unique.

An approach is chosen identical to Einstein's conventions. It *defines* the two-way sound velocity, denoted by c to be equal for all reference frames. Assume then a rod of length $\overline{AB}$ and a sound wave emitted from a point A at a time t_A travelling to B where it is reflected and arrives at A at $t_{A'}$; then the two-way sound velocity c can be defined by $c := 2\overline{AB}/(t_{A'} - t_A)$. Combining (3.6) with the convention of the invariance of the sound speed yields the transformation of space scales:

$$\frac{\mathrm{d}x_2}{\mathrm{d}x_1} = \frac{\omega_1}{\omega_2} . \tag{3.7}$$

Although no choice for the relation between energy parameters in two different frames has been made yet, it is obvious from (3.6) and (3.7) that it will be of greatest importance to the transformation properties of the scales. So far, ω_1 and ω_2 are arbitrary parameters. We now compare these two parameters in order to obtain the specific form of time dilation and rod contraction.

The clock under consideration, that is the one at rest relative to the second frame S_2, moves with a velocity v_1 (measured in S_1) relative to S_1. The frame S_1 itself is at rest relative to the medium. In this configuration, ω_1 can be identified with the dispersion relation

$$\omega_1 = \omega(p) . \tag{3.8}$$

It is not evident at first glance what value should be assigned to ω_2. One possibility would be to accept a preferred frame of reference, namely S_1, and set

$$\omega_2 = \omega(p)\,. \tag{3.9}$$

This would result in Galileian-type transformation laws, as can be seen from (3.6) and (3.7), if one requires the description (3.5) of the clock to be covariant (the same for all frames of reference).

However, if it is not easy or impossible to distinguish between frames of reference by measuring their motion relative to the medium, the value $\omega(p = 0)$ for zero velocity may be assigned to ω_2. More precisely, this can be formulated in the following way: if an object at rest in the frame of reference S_1 with energy ε_1 is transferred to another frame of reference S_2 so that it is at rest there, an energy

$$\varepsilon_2 := \varepsilon_1 \tag{3.10}$$

is assigned to it in the second frame. This is an « evident », although not unique way of camparing energy scales in different frames of one and the same medium with one another. It yields the *most symmetric* description. If we require general covariance of the physical laws, eqs. (3.4) are fulfilled identically, and the dilatation laws are obtained immediately by insertion in (3.6) and (3.7):

$$\frac{\mathrm{d}t_2}{\mathrm{d}t_1} = \frac{\mathrm{d}x_2}{\mathrm{d}x_1} = \frac{\omega(p)}{\omega(0)}\,. \tag{3.11}$$

These dilatation laws do not necessarily reproduce Lorentz invariance; they are far more general and can be applied to arbitrary media with very different dispersion relations. They represent one major result of this work. Coming back to the discussion of operational-parameter descriptions, we note that, even if there would exist a very strong evidence for a non–Lorentz-invariant medium in some exotic limit, or if there would be empirical evidence of a preferred frame of reference (such as the frame at rest with respect to the cosmic background radiation), there is no need to spoil the convenient symmetric notation. However, one then should always bear in mind that, although the formalism is written symmetrical, the phenomenology is not.

Generally, dispersion relations may depend not only on momentum, but for inhomogeneous media also on space co-ordinates of arbitrary dimensions and for nonlinear forces on the square of the amplitudes as well as on other parameters such as a fundamental length. In these cases considerations are quite subtle, since the sound velocity may depend on the frequency and hence scales would depend on the frequency chosen for synchronization.

C) *A model medium.* One of the more interesting systems and one which has been extensively discussed in the literature (17) is a three-dimensional continuous, homogeneous and isotropic field with its points or constituents coupled linearly to the equilibrium position and neighbours. It will serve as a perfect medium model, reproducing Lorentz-type transformation laws. There is a wide class of physical problems, where a medium can be linearized in that way: for example, acoustic phonons and other quasi-particles in solid-state and many-particle physics, or so-called cellular automata. The dispersion relation is given by $\omega(p) = |(cp/k)^2 + \omega_0^2|^{\frac{1}{2}}$, where c and ω_0 are two constants proportional to spring strengths and k is the minimal action of (2.7). Again a configuration is considered in which a clock travels with velocity $v_1 = \mathrm{d}\omega_1(p)/\mathrm{d}p$ with respect to the medium. In this case it is possible to identify $\omega_1(p)$ with $\omega(p)$, and the following time dilatation is obtained:

$$\frac{\mathrm{d}t_2}{\mathrm{d}t_1} = \left[1 - \left(\frac{v_1}{c}\right)^2\right]^{-\frac{1}{2}}, \tag{3.12}$$

and, since v_1 equals $-v_2$, where v_2 is the velocity of S_1 measured from as S_2 as a consequence of (3.11),

$$(\mathrm{d}t_1)^2 = (\mathrm{d}t_2)^2 - \frac{1}{c^2}(\mathrm{d}x_2)^2. \tag{3.13}$$

Since this relation holds true for all frames S_2 for arbitrary velocities v, the right-hand side of (3.13) is an invariant, which implies Lorentz-type transformation laws. I want to emphasize that, like all transformation laws of space and time scales, this result depends on two independent inputs: i) conventions such as clock synchronization, which are more or less arbitrary (16), and ii) the structure of the system with its specific dispersion relation. Which operational-parameter description we choose is a question of economy, symmetry and so on and it is quite subjective. If we would indeed live in a three-dimensional medium of above type, we would very likely end up with a Lorentz-invariant formalism, since this parameter description is more symmetric and formally appealing. However, in order to proceed, we would be forced to accept the notion of a medium since this opens up doors to new phenomenology.

4. – How to detect the medium.

If the vacuum is thought of in more realistic terms as a ground state of a quantum medium, then the immediate question is: what can be the use of such a notion? Despite other more subjective reasons, the ultimate goal should

(17) E. M. Henley and W. Thirring: *Elementary Quantum Field Theory* (McGraw-Hill, New York, N. Y., 1962).

be the experimental detection of this medium, which might subsequently yield possibilities of transforming the vacuum into new phases with very different characteristics (maximum propagation speed higher than the speed of light, higher action resolution and that like). Although these are mere speculations at present, they represent a progressive scientific program, which has to be tested experimentally (9).

This rather utopian goal has to be initiated by a careful study of possible indicators for such a medium. In what follows I shall give a brief list of operational parameters which have the potentiality to indicate a medium:

i) *Granular structure of space-time*: its detection will very much depend on the type of granularity involved. The simplest case of an energy-momentum cut-off corresponding to a fundamental length has recently been discussed by SHUPE (18). But much more complex and subtle irregularities may occur (19,20) yielding a fractal space-time with associated Hausdorff dimension not equal to four (21). Since this has an impact on all quantum-mechanical matrix elements, in particular sensitive radiative corrections such as the anomalous magnetic moment, this noninteger Hausdorff dimension would operationally be attainable: with present accuracy, it is $4-(5.3\pm 2.5)\cdot 10^{-7}$.

ii) The *cosmic background radiation* represents the possibility of existence of an absolute frame of reference, but it is not a forcing reason to accept a quantum medium, since this only concerns the initial condition of the world rather than the structure of the theory and the associated equations of motion. However, if a quantum medium should eventually be detected, it will be presented as the first decisive indication in future physics textbooks.

iii) *Deviations from the theory of special relativity*: since its introduction the theory of special relativity has developed into a reference theory, almost sacrosanct to objections. Despite prevailing discussions about the detectability of the one-way velocity of light, there is no evidence of failure. However, some deviations from the ideal dispersion relation $E(p) = [(cp)^2 + (m_0 c^2)^2]^{\frac{1}{2}}$ due to nonlinearities of the quantum medium may eventually occur at very high energies, but there seems to be no scale to tell what high in this respect means. For a detailed analysis of the present experimental tests of special relativity

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(18) M. A. SHUPE: *Am. J. Phys.*, **53**, 122 (1985).
(19) P. C. W. DAVIES: in *Quantum Gravity 2*, edited by C. J. ISHAM, E. FISCHBACH, M. P. HAUGAN, D. TADIĆ and H. Y. CHENG (Clarendon Press, Oxford, 1981), p. 207.
(20) C. J. ISHAM: in *Quantum Theory of Gravity*, edited by ST. M. CHRISTENSEN (Adam Hilger Ltd., Bristol, 1979).
(21) A. ZEILINGER and K. SVOZIL: *Phys. Rev. Lett.*, **54**, 2553 (1985); *J. Mod. Phys.*, in print.

and trends, the reader is referred to ref. ([22-24]). KIRZHNITS and CHECHIN, for instance, infer from the ultraenergetic end (10^{10} GeV) of cosmic proton spectra, a deviation from the relativistic dispersion relation ([25]).

iv) There may exist a large number of further effects that have not been proposed yet, but will shed new insight into the theory of the vacuum structure. Presently it seems unlikely that this will happen soon, but one never knows. In this context experiments to test fundamental properties of space-time and quantum structure in extreme limits are necessary.

In conclusion, it can be said that, since we are not able to look at our world from above and utilize an extrinsic-parameter description, we are forced to use operational-parameter descriptions (presently, there is not even a quasi-extrinsic–parameter description conceivable). It has been shown that from a phenomenological point of view (with present accuracy) a medium with a particular type of dispersion relation yields an operational-parameter description for space-time and energy identical to parameter descriptions obtained from the abstract assumption of Lorentz covariance. This fact alone would not justify attention. However, the Lorentz-invariant quantum medium yields a phenomenology not predictable by the special theory of relativity.

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I am indebted to A. ZEILINGER for many helpful discussions, as well as to G. GRÖSSING for pointing out the importance of the cybernetic approach. This work was supported by the Austrian Ministry for Science and Research, project number 19.153/3-26/85, « Studium neuer Konzeptionen der Systemevolution ».

([22]) A. K. A. MACIEL and J. TIOMNO: *Phys. Rev. Lett.*, **55**, 143 (1985).
([23]) E. FISCHBACH, M. P. HAUGAN, D. TADIĆ and H.-Y. CHENG: *Phys. Rev. D*, **32**, 154 (1985).
([24]) D. W. MACARTHUR: *Phys. Rev. A*, **33**, 1 (1986).
([25]) D. A. KIRZHNITS and V. A. CHECHIN: *Sov. J. Nucl. Phys.*, **15**, 585 (1972).

● RIASSUNTO (*)

Si discute il concetto di parametri operazionali. I parametri operazionali sono confrontati con variabili estrinseche o nascoste. Si mostra che, applicando le convenzioni di sincronizzazione di Einstein, le proprietà di trasformazione dei parametri di spazio e tempo dipendono dalla relazione di dispersione (energia impulso) del sistema. In particolare, con forze non lineari che agiscono in un mezzo, le leggi di trasformazione non devono essere necessariamente della forma di Lorentz.

(*) *Traduzione a cura della Redazione.*

Операторные концепции и законы преобразования пространственно-временных координат в произвольной диспергирующей среде.

Резюме (*). — Обсуждается концепция операторных параметров. Операторные параметры сравниваются с несобственными или скрытыми переменными. Применяя условия сихронизации Эйнштейна, показывается, что свойства преобразования пространственных и временных операторных параметров зависят от дисперсионного соотношения системы (связь энергии и импульса). В частности, в случае нелинейных спл, действующих в среде, законы преобразования не обязательно должны иметь лоренцеву форму.

(*) *Переведено редакцией.*

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11 Dicembre 1986
Il Nuovo Cimento
Serie 11, Vol. **96** B, pag. 127-139