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Possible Test of the Weak Boson Self-Coupling Below the W^+W^--Threshold

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Abstract. The weak boson self-coupling contributes to the decay $Z^0 \to W^\pm X^\mp$. It is argued how this contribution provides a possible, though experimentally difficult test of the gauge type nature of the electroweak interactions below the W^+W^- threshold.

It is quite commonly accepted, that the prediction and (hopeful) discovery of the intermediate bosons Z^0 and $W^\pm$ (at 90 and 79,5 GeV respectively) forms the next crucial test of standard unified electroweak theory. However, in order to establish the gauge theoretical nature of these interactions, it is necessary to measure the strength of the direct boson-boson coupling [1] predicted to be

$$\tfrac{1}{4}\mathbf{A}_{\mu\nu}\mathbf{A}^{\mu\nu} - \tfrac{1}{2}(\partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu)\partial^\mu \mathbf{A}^\nu = g[\mathbf{A}_\mu \times \mathbf{A}_\nu]\partial^\mu \mathbf{A}^\nu + \frac{g^2}{4}\{(\mathbf{A}_\mu \mathbf{A}^\mu)^2 - (\mathbf{A}_\mu \mathbf{A}_\nu)(\mathbf{A}^\mu \mathbf{A}^\nu)\}, \quad (1)$$

where

$$\mathbf{A}_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu + g[\mathbf{A}_\mu \times \mathbf{A}_\nu]. \quad (2)$$

The relation of the weak isovector $\mathbf{A}_\mu$ to the physical bosons is the usual,

$$A_1^\mu + iA_2^\mu = \sqrt{2}W^\mu$$
$$A_3^\mu = \cos\theta_W Z^\mu - \sin\theta_W A^\mu, \quad (3)$$

where A^μ is the photon and θ_W the weak mixing angle. Furthermore

$$e = g\sin\theta_W \quad (4)$$

so that the self-coupling predicted by gauge invariance is completely determined by e and θ_W.

It has been noted quite early [2] that not only does gauge invariance (with spontaneous symmetry breaking) lead to a renormalizable theory, but that acceptable high energy behaviour requires gauge couplings of the type of (1) to insure cancellation of the most divergent contributions. Hence it is quite obvious that the ideal test for coupling (1) is the process

$$e^+ + e^- \to W^+ + W^- \quad (5)$$

because the cross-section is quite sensitive to the precise value of the 3 boson vertex. But process (5) requires very high energy, probably not available at the first stage of LEP. Below this threshold, the vertex can – at least in principle – be detected by means of processes of the type

$$Z^0 \to W^+ + X, \quad (6)$$

where X can be any suitable state. Processes of this kind have been discussed in the literature [3, 4] as possible sources of W-bosons below the W^+W^- threshold. Here, we want to emphasize the possible determination of the gauge vertex (1) by means of process (6).

The trouble is that process (6) can proceed not only via the three boson vertex (see Fig. 1a) but also via conventional (non-gauge type) diagrams (see Fig. 1b). In fact, in [3], only diagrams of the type Fig. 1b have been used to estimate single W production. In case X is a leptonic state, Fig. 1b has to be replaced by Fig. 2. Also in the case of (inclusive) hadronic processes, we may use graphs of the type of Fig. 2 with leptons replaced by quarks ignoring the hadronization, to obtain estimates.

Let us first consider the leptonic X-state, i.e. the process

$$Z^0 \to W^+ + l^- + \bar{\nu}_l. \quad (7)$$

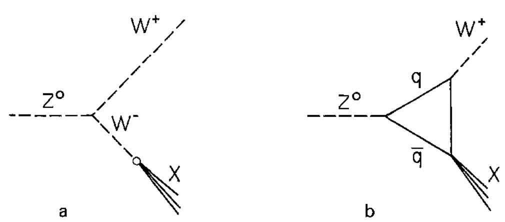

Fig. 1a and b. Contributing diagrams for process (6)

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Contributions of the graph Fig. 1a, the graphs of Fig. 2 and of the interference of the gauge type with the conventional diagrams are

$$\Gamma(Z^0 \to W^+ + l^- + \bar{\nu}_l) = \Gamma_{\text{gauge}} + \Gamma_{\text{conv}} + \Gamma_{\text{int}} \tag{8}$$

with (neglecting the lepton mass in the matrix elements)

$$\Gamma_{\text{gauge}} = \frac{g^4}{48\pi^3} \int_{m_W}^{E_0} \frac{dE}{m_Z} \frac{(E^2 - m_W^2)^{3/2}}{(m_Z - 2E)^2} \cdot \left[(1 + \cos^2\theta_W)\left(1 + \cos^2\theta_W - \frac{2E}{m_Z}\right) + \tfrac{2}{3}\cos^2\theta_W + \frac{E^2}{3m_Z^2} \right], \tag{9a}$$

$$\begin{aligned}\Gamma_{\text{conv}} = \frac{g^4}{384\pi^3} \int_{m_W}^{E_0} dE \Bigg\{ & \frac{2(E^2 - m_W^2)^{1/2}}{m_Z} \cdot \Bigg[\cos^4\theta_W - \tfrac{7}{3}\cos^2\theta_W + 5 - \frac{2}{\cos^2\theta_W} \\ & - \frac{2E}{m_Z}(1 + \cos^2\theta_W) + \frac{E^2}{3m_Z^2} \Bigg] + \frac{1}{\cos^2\theta_W} \ln \left| \frac{E + (E^2 - m_W^2)^{1/2}}{E - (E^2 - m_W^2)^{1/2}} \right| \\ & \cdot \Bigg[\frac{2E}{m_Z}(\sin^4\theta_W + \cos^4\theta_W) - \frac{m_Z}{E}(1 + \cos^2\theta_W) \\ & \cdot (1 - 2\sin^2\theta_W)\left(1 + \cos^2\theta_W - \frac{2E}{m_Z}\right) \Bigg] \Bigg\}, \end{aligned} \tag{9b}$$

$$\begin{aligned}\Gamma_{\text{int}} = \frac{g^4}{96\pi^3} m_Z \int_{m_W}^{E_0} \frac{dE}{m_Z - 2E} \Bigg\{ & \frac{2E(E^2 - m_W^2)^{1/2}}{m_Z^2} \cdot \Bigg[(1 + \cos^2\theta_W)\left(1 + \cos^2\theta_W - \frac{2E}{m_Z}\right) \\ & + \tfrac{2}{3}\cos^2\theta_W + \frac{E^2}{3m_Z^2} \Bigg] - \cos^2\theta_W \ln \left| \frac{E + (E^2 - m_W^2)^{1/2}}{E - (E^2 - m_W^2)^{1/2}} \right| \\ & \cdot \Bigg[\cos^2\theta_W + (1 + \cos^2\theta_W)\left(1 + \cos^2\theta_W - \frac{2E}{m_Z}\right) \Bigg] \Bigg\}, \end{aligned} \tag{9c}$$

where E is the energy of the W^+ and E_0 its maximal value.

For $l = e$, and $\sin^2\theta_W = 0.22$ the conventional contribution alone yields 11 eV, whereas the total contribution, including the 3 boson vertex part, gives 17 eV. Thus the change due to the 3 boson vertex is about 40%. In itself, this is encouraging, but unfortunately it has to be compared to the total width of the Z^0-boson of 2–3 GeV [5]. The first reaction might be to drop the investigation altogether. To avoid this, we would like to remind the reader of a similar situation in history: The most crucial test for the conserved vector current

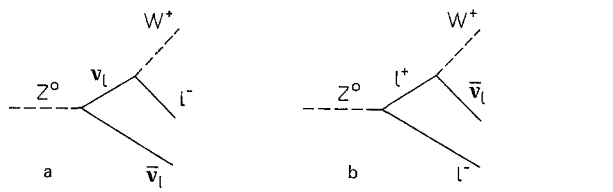

Fig. 2a and b. Conventional diagrams for process (7)

theory in the early 1960's turned out to be the decay

$$\pi^+ \to \pi^0 + e^+ + \nu_e, \tag{10}$$

whose branching ratio was predicted to be $1.05 \cdot 10^{-8}$ (including radiative corrections). In the old days, this was certainly as prohibitive for π^- decay as it is now for Z^0-decay; nevertheless, in view of the importance of a clear test of the conserved vector current theory, experiments were driven [6] to the breathtaking accuracy of $(1.023 \pm 0.069) \cdot 10^{-8}$. We would like to reiterate, that the existence of Z^0 and $W^\pm$ at the predicted masses, though beautiful in itself, does not verify the gauge nature of interaction until the 3 boson vertex strength is measured!

In the hadronic case, computations are more cumbersome. Details will be given elsewhere [7]. The main result is again a comparison of width with and without the 3 boson vertex contribution. They are given in Table 1.

Table 1. Decay width contributions for some characteristic hadronic final states X in process (6)

X	Γ_{gauge} (eV)	Γ_{conv} (eV)	Γ_{int} (eV)	Γ_{tot} (eV)
π^-	1.04	0.25	− 1.0	0.29
ϱ^-	2.80	1.98	− 2.73	2.05
A_1^-	1.33	1.5	− 1.28	1.55
$\sum_{\text{color}} \bar{u}d$	150.0	28.5	−131.1	47.4

Let us conclude by encouraging an experiment which is no doubt very difficult, but seems to be the crucial test of the gauge nature of electroweak interactions until energies well above the W^+W^- threshold (i.e. 159 GeV) are available.

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