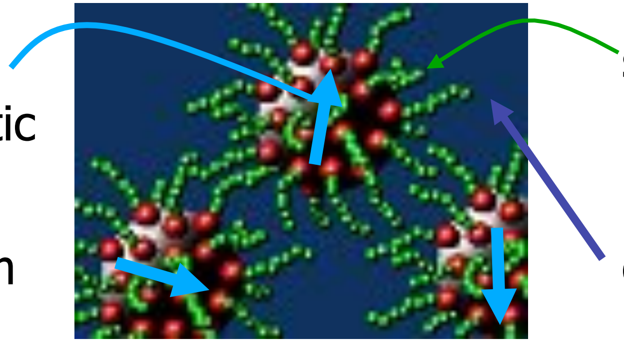
Rotational Brownian motion:

An application to noise-induced effects in ferrofluids

Ferrofluids

permanent magnetic dipoles

particle size ~10nm

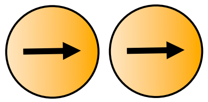


steric stabilization

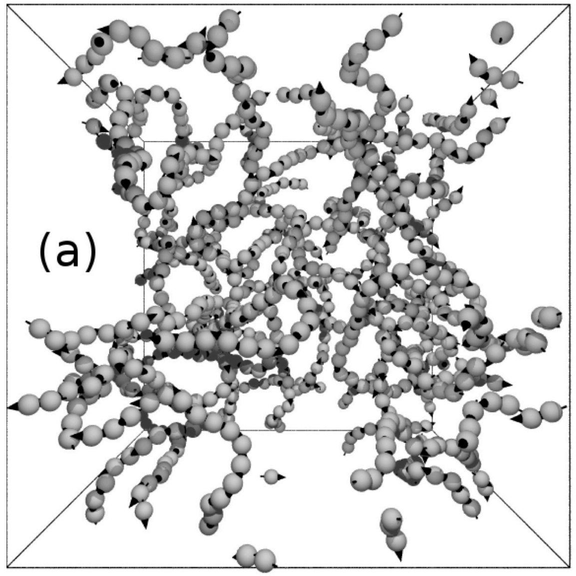
carrier liquid

magnetic dipole-dipole interaction:

$$u_{DD}(12) = \frac{\mu^2}{r^3}\left(\underline{\hat{\mu}}_1 \cdot \underline{\hat{\mu}}_2 - 3(\underline{\hat{\mu}}_1 \cdot \underline{\hat{r}})(\underline{\hat{\mu}}_2 \cdot \underline{\hat{r}})\right)$$

preferred configuration: 

➡ particles tend to form chains even without fields!



MD simulations: Jordanovic, Jaeger, Klapp, Phys Rev Lett (2011)

Consider one magnetic dipole in an oscillatory field

$$\underline{B} = \left(B_x, B_y(t), 0\right)$$

$$B_y(t) = B_y^0\left(\cos(\omega t) + c\sin(2\omega t + \delta)\right)$$

A. Engel, H. W. Mueller, P. Reimann, A. Jung,
Phys. Rev. Lett. **91** (2003) 060602

Overdamped Langevin equation (only rotations of unit dipole vector $\underline{e}_i$ are considered!)

$$\zeta_R \dot{\underline{e}}_i = (\underline{e}_i \times \underline{B}^{external}(t) + \underline{T}_i^{ran}) \times \underline{e}_i$$

Restrict orientational motion to 2D: consider phase angle

$$\frac{d\phi}{dt} = \partial_\phi (B_x \cos\phi + B_y(t)\sin\phi) + \xi^{ran}(t)$$

Consider one magnetic dipole in an oscillatory field

$$\underline{B} = \left(B_x, B_y(t), 0\right)$$

$$B_y(t) = B_y^0\left(\cos(\omega t) + c\sin(2\omega t + \delta)\right)$$

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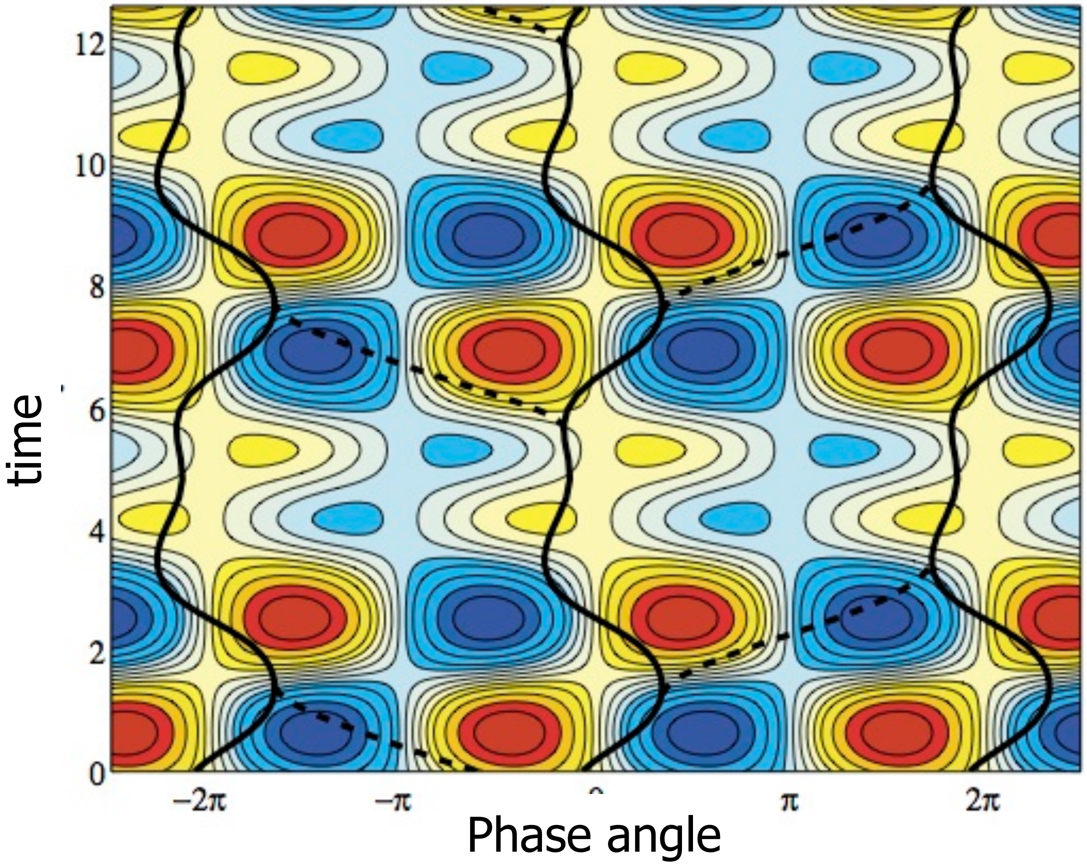

deterministic trajectories and noise-induced "jumps"

Consider one magnetic dipole in an oscillatory field

$$\underline{B} = \left(B_x, B_y(t), 0\right)$$

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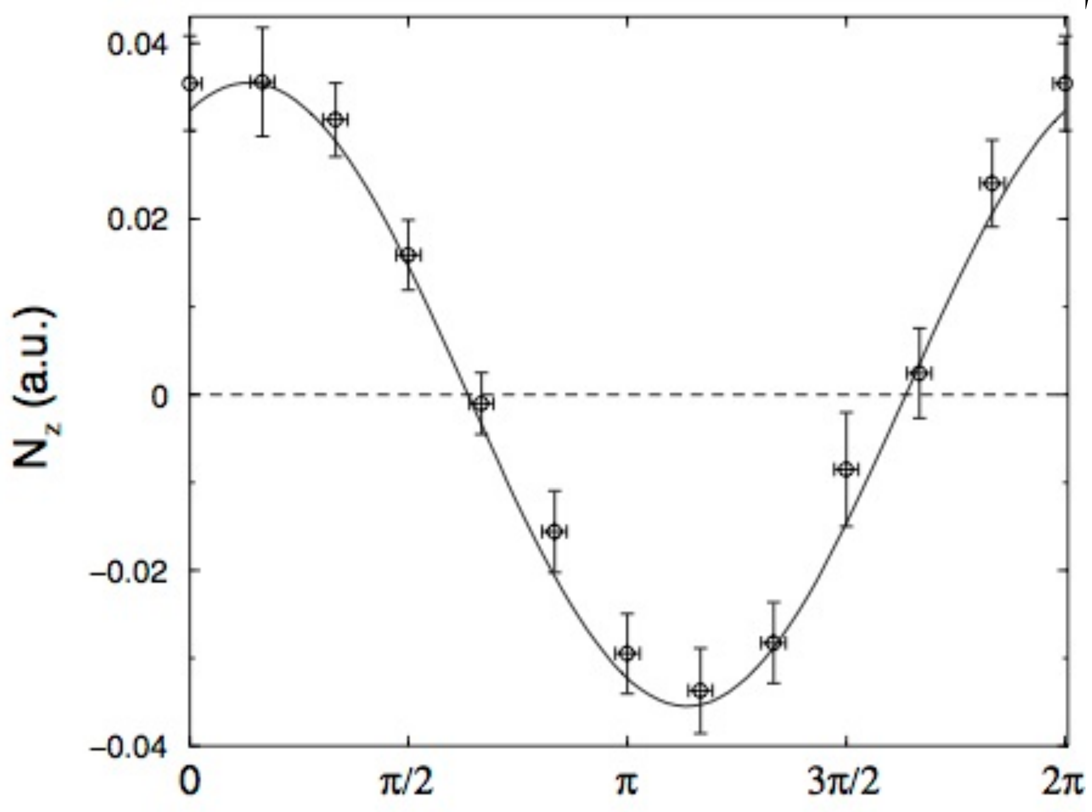


Phase shift of the external field

non-vanishing torque – although the field does not rotate !!

Computer simulations

Sebastian Jaeger

Model: Soft spheres with permanent dipoles

$$u^{DSS}(12) = 4\varepsilon\left(\frac{\sigma}{r}\right)^{12} + \frac{\mu^2}{r^3}\left(\underline{\hat{\mu}}_1 \cdot \underline{\hat{\mu}}_2 - 3(\underline{\hat{\mu}}_1 \cdot \underline{\hat{r}})(\underline{\hat{\mu}}_2 \cdot \underline{\hat{r}})\right)$$

Equations of motion (overdamped Langevin):

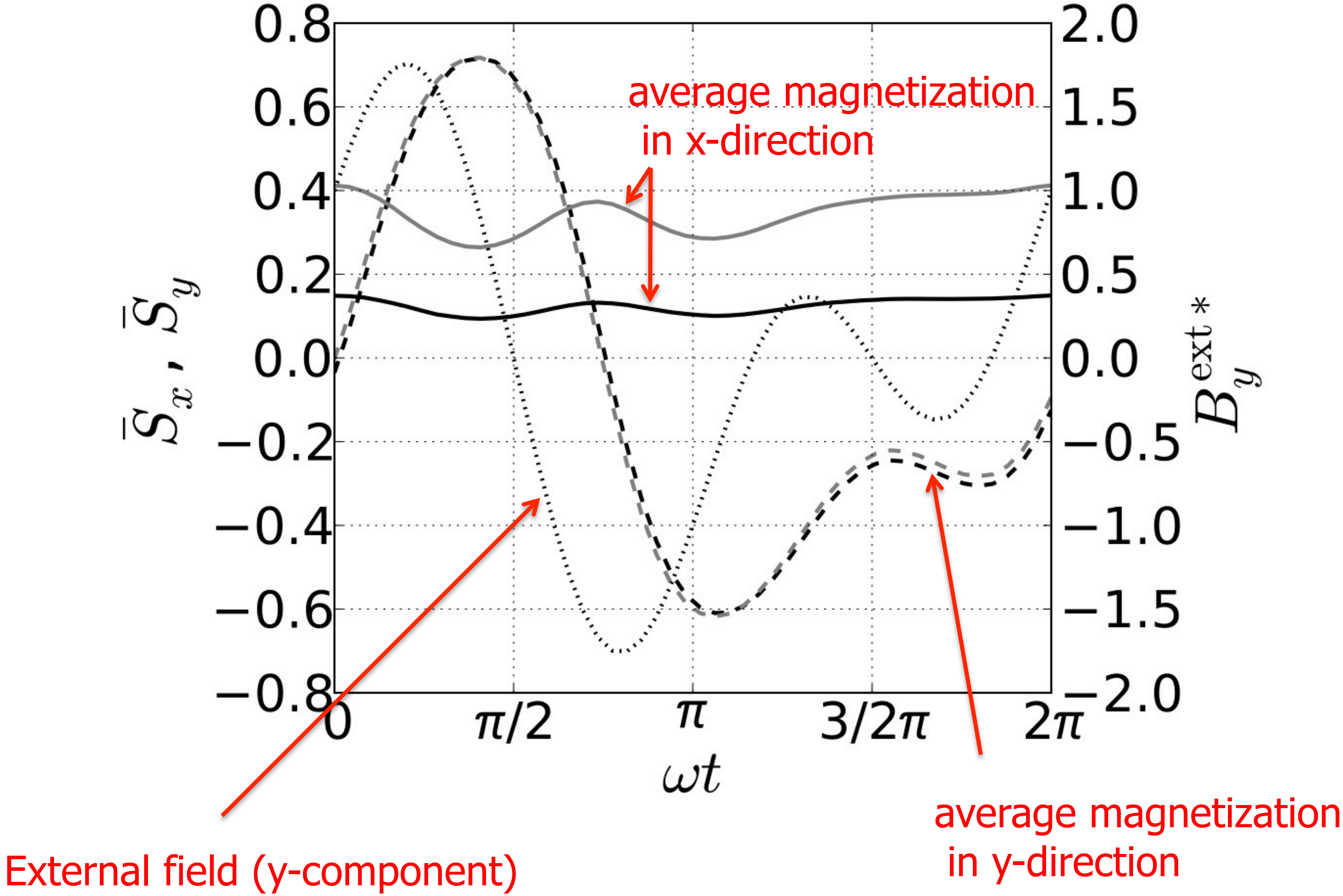
$$\zeta_T \dot{\underline{r}}_i = \underline{F}_i^{DSS} + \underline{F}_i^{ran}$$

$$\zeta_R \dot{\underline{e}}_i = \left(\underline{T}_i^{DSS} + \underline{\mu}_i \times \underline{B}^{external}(t) + \underline{T}_i^{ran} \right) \times \underline{e}_i$$

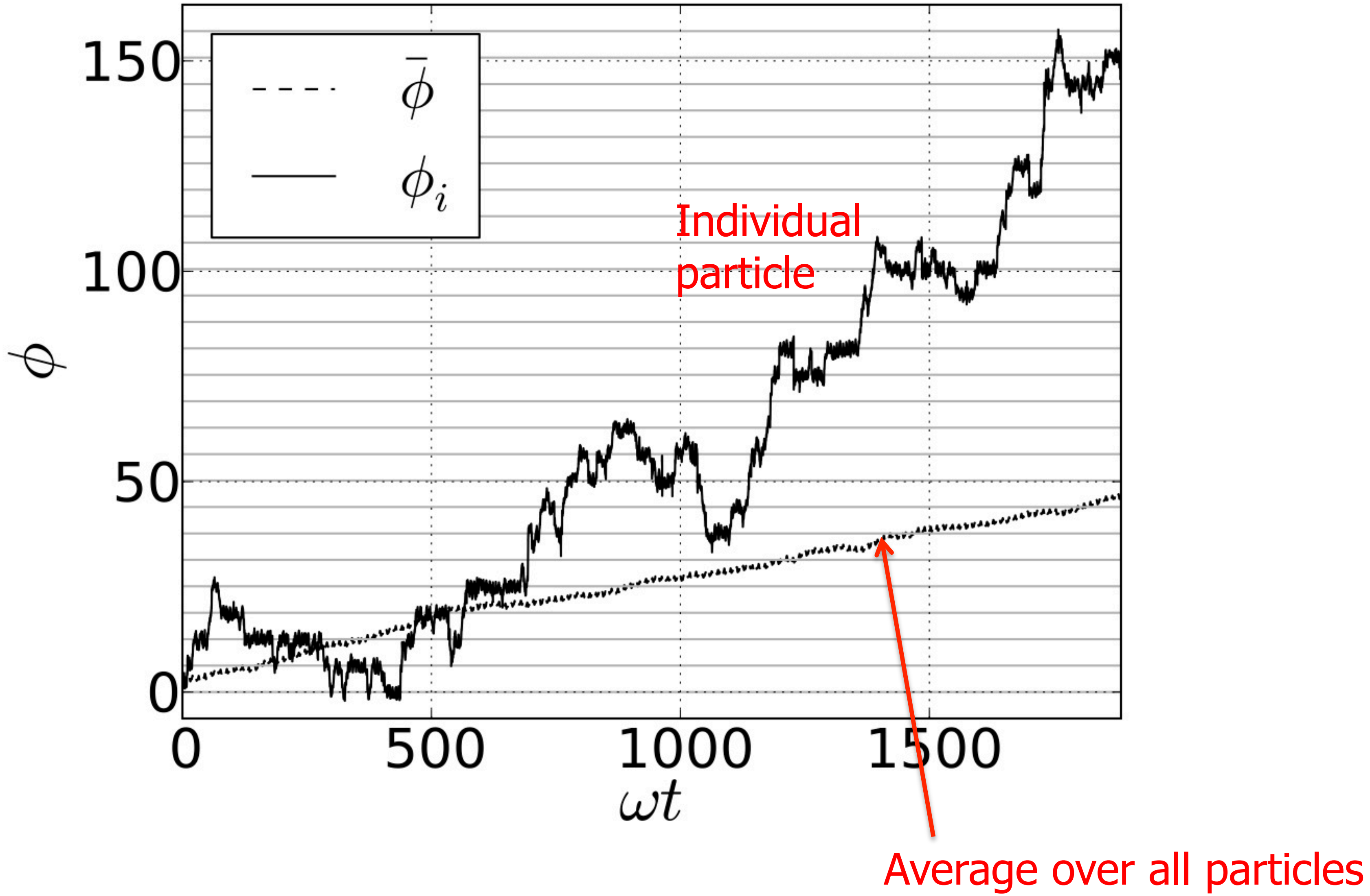
3D system, rotations AND translations!

Sebastian Jaeger and Sabine H.L. Klapp, arXiv 1210.3479 (2012)

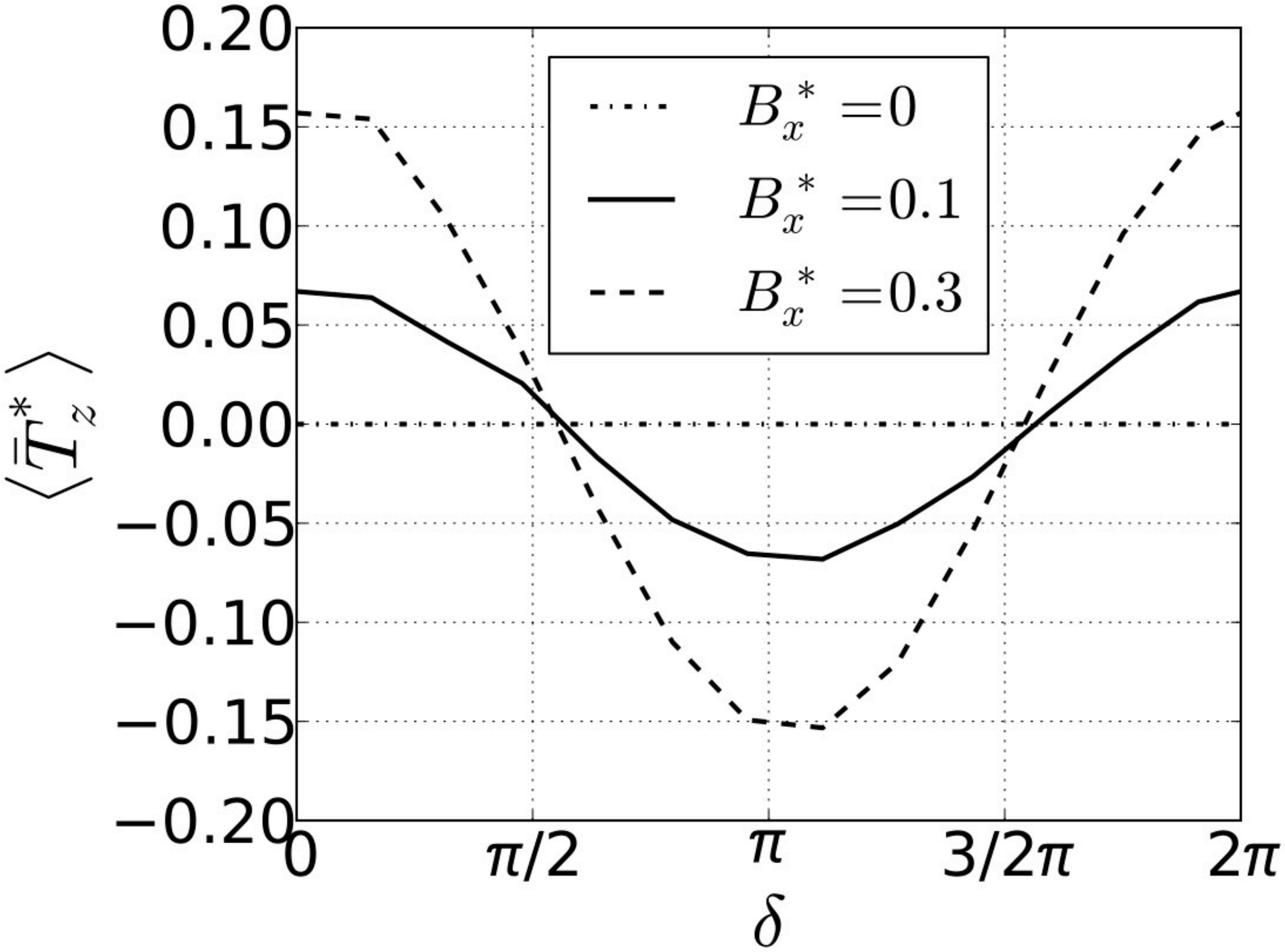
Simulation results



Trajectories of the phase angle

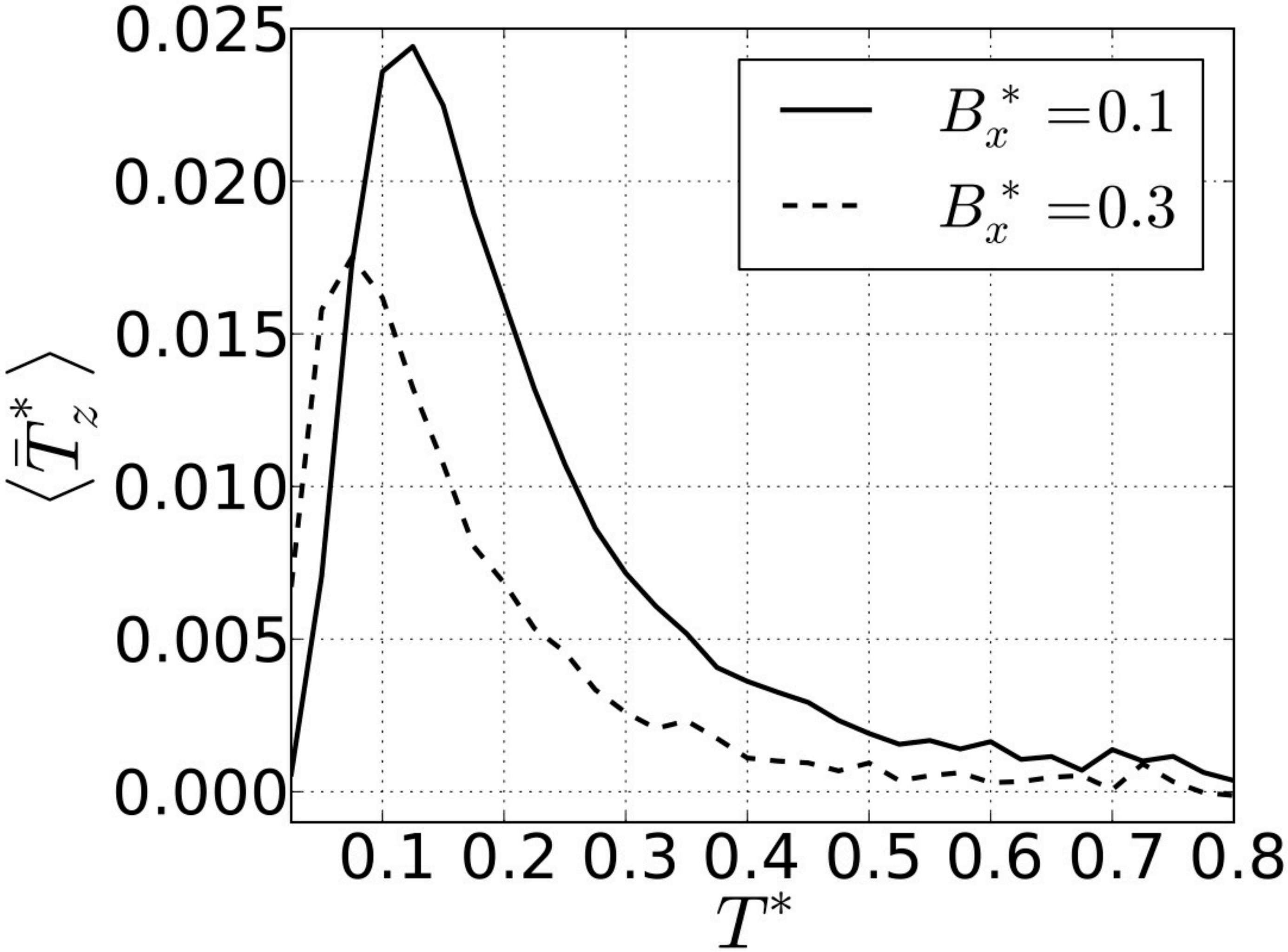


Net torque



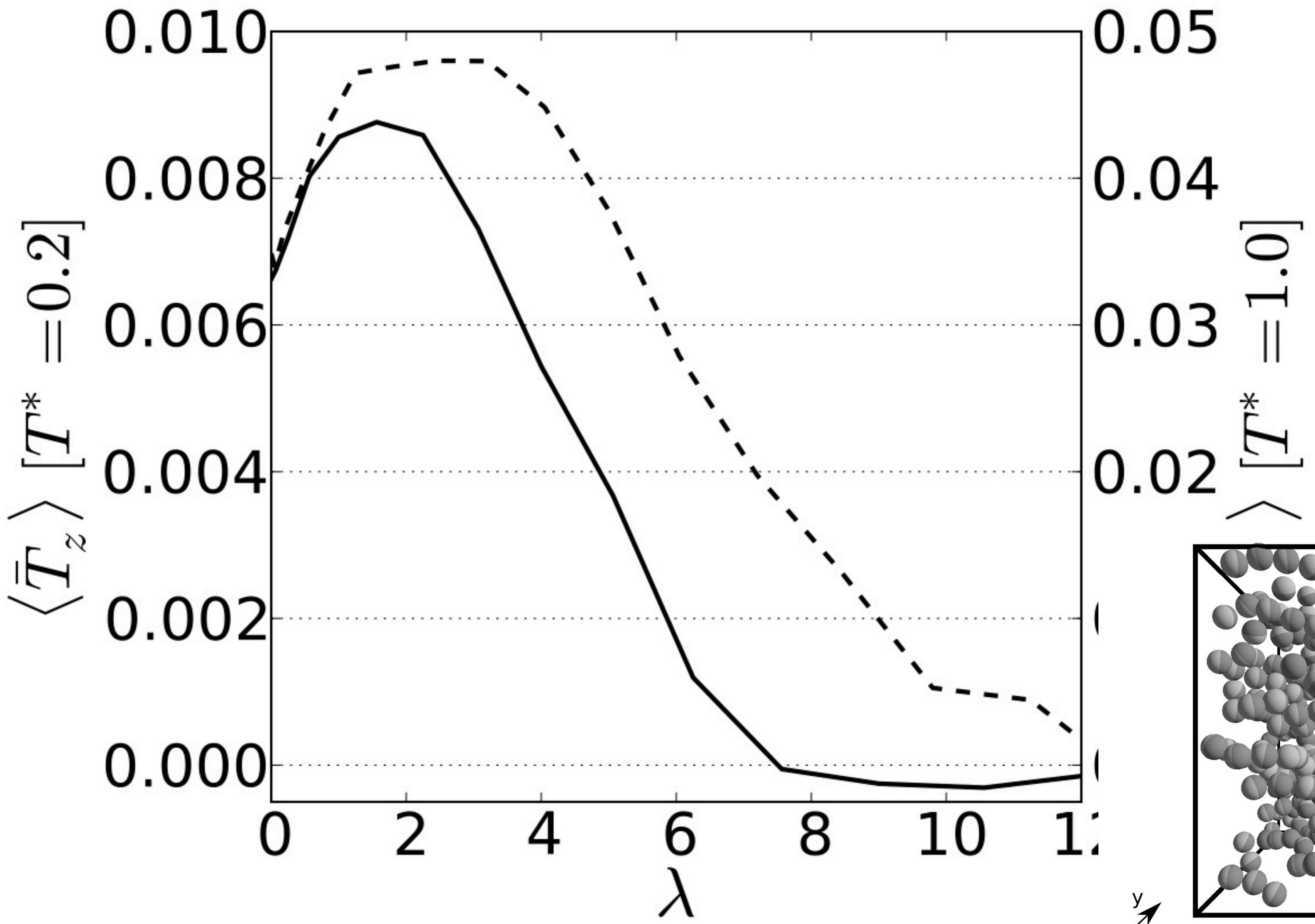
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
$\langle \bar{T}_z^* \rangle$
0
π/2
π
3/2π
2π
δ
$B_x^* = 0$
$B_x^* = 0.1$
$B_x^* = 0.3$

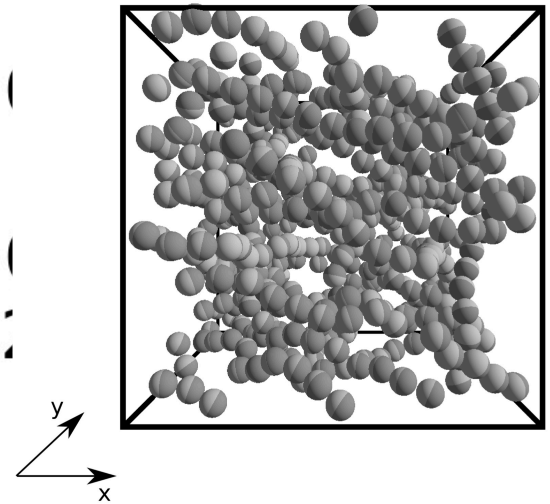
Influence of noise on the net torque



Here: Temperature = noise strength

Impact of dipolar (and repulsive) interactions between the particles





Dipolar coupling strength