Extrinsic-intrinsic concept & complementarity

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1 Introduction

Epistemological, the *intrinsic/extrinsic* concept, or, by another naming [1, 2], the *endophysics/exophysics* concept, is related to the question of how a mathematical or a logical or an algorithmic universe is perceived from within/from the outside. The physical universe, by definition, can be perceived from within only.

Extrinsic or *exophysical* perception can be conceived as a hierarchical process, in which the system under observation and the experimenter form a two-level hierarchy. The system is laid out and the experimenter peeps at every relevant feature of it without changing it. The restricted entanglement between the system and the experimenter can be represented by a one-way information flow from the system to the experimenter; the system is not affected by the experimenter's actions. (Logicians might prefer the term *meta* over *exo*.)

Intrinsic or *endophysical* perception can be conceived as a non-hierarchical effort. The experimenter is part of the universe under observation. Experiments use devices and procedures which are realisable by internal resources, i.e., from within the universe. The total integration of the experimenter in the observed system can be represented by a two-way information flow, where "measurement apparatus" and "observed entity" are interchangeable and any distinction between them is merely a matter of intent and convention. Endophysics is limited by the self-referential character of any measurement. An intrinsic measurement can often be related to the paradoxical attempt to obtain the "true" value of an observable while — through interaction — it causes "disturbances" of the entity to be measured, thereby changing its state. Among other questions one may ask, *"what kind of experiments are intrinsically operational and what type of theories will be intrinsically reasonable?"*

Imagine, for example, some artificial intelligence living in a (hermetic) cyberspace. This agent might develop a "natural science" by performing experiments and developing theories. It is tempting to speculate that also a figure in a novel, imagined by the poet and the reader, is such an agent.

Since in cyberspace only *syntactic* structures are relevant, one might wonder if concerns of this agent about its "hardware basis," e.g., whether it is "made of" billard balls, electric circuits, mechanical relays or nerve cells, are mystic or even possible (cf. H. Putnam's brain-in-a-tank analysis [4]). I dont think this is necessarily so, in particular if the agent could influence some features of this hardware basis. One example is a hardware damage caused by certain computer viruses by "heating up" computer components such as storage or processors. I would like to call this type of "back-reaction" of a virtual reality on its computing agent *"virtual backflow interception"* (VBI). Intrinsic phenomenologically, VBI could manifest itself by some violation of a "superselection rule;" i.e., by some virtual phenomenon which violates the fundamental laws of a virtual reality, such as symmetry & conservation principles.

No attempt is made here to (re-)write a comprehensive history of related concepts; but a few hallmarks are mentioned without claim of completeness. Historically, Archimedes conceived *"points outside the world, from which one could move the earth."* Archimedes' use of "points outside the world" was in a mechanical rather than in a metatheoretical context: he claimed to be able to move any given weight by any given force, however small. The 18'th century physicist B. J. Boscovich realised that it is not possible to measure motions or transformations if the whole world, including all measurement apparata and observers therein, becomes equally affected by these motions or transformations (cf. O. E. Rössler [2], p. 143). Fiction writers informally elaborated consequences of intrinsic perception. E. A. Abbot's *Flatland* describes the life of two- and onedimensional creatures and their confrontation with higher dimensional phenomena. The *Freiherr von Münchhausen* rescued himself from a swamp by dragging himself out by his own hair. Among contemporary science fiction authors, D. F. Galouye's *Simulacron Three* and St. Lem's *Non Serviam* study some aspects of artificial intelligence in what could be called "cyberspaces." Media artists such as Peter Weibel create "virtual realities" or "cyberspaces" and are particulary concerned about the *interface* between "reality" and "virtual reality," both practically and philosophically. Finally, by outperforming television & computer games, commercial "virtual reality" products might become very big business. From these examples it can be seen that concepts related to intrinsic perception may become fruitful for physics, the computer sciences and art as well.

Already in 1950 (19 years after the publication of Gödel's incompleteness theorems), K. Popper has questioned the completeness of self-referential perception of "mechanic" computing devices [5]. Popper uses techniques similar to Zeno's paradox (which he calls "paradox of Tristram Shandy") and "Gödelian sentences" to argue for a kind of "intrinsic indeterminism."

In a pioneering study on the theory of (finite) automata, E. F. Moore has presented *Gedanken-experiments on sequential machines* [6]. There, E. F. Moore investigated automata featuring, at least to some extend, similarities to the quantum mechanical uncertainty principle. In the book *Regular Algebra and Finite Machines* [30], J. H. Conway has developed these ideas further from a formal point of view without relating them to physical applications. Probably the best review of experiments on Moore-type automata can be found in W. Brauer's book *Automatentheorie* [31] (in German).

D. Finkelstein [32, 33] has considered Moore's findings from a more physical point of view, introducing an "experimental logic of automata" and the term *"computational complementarity."* An illuminating account on endophysics topics can be found in O. E. Rössler's article on *Endophysics* [1], as well as in his book *Endophysics* (in German)

[2]; O. E. Rössler is a major driving force in this area. Also H. Primas has considered endophysical and exophysical descriptions in various contexts [7].

The terms *"intrinsic"* and *"extrinsic"* appear in the author's studies on intrinsic time scales in arbitrary dispersive media [8, 9, 10]. There, the intrinsic-extrinsic concept has been re-invented (probably for the 100'th time, and, I solemnly swear) independently. It is argued that, depending on dispersion relations, creatures in a "dispersive medium" would develop a theory of coordinate transformation very similar to relativity theory. Another proposal by the author was to consider a new type of "dimensional regularisation" by assuming that the space-time support of (quantum mechanical) fields is a fractal [11]. In this approach one considers a fractal space-time of Hausdorff dimension $D = 4 - \varepsilon$, with $\varepsilon \ll 1$, which is embedded in a space of higher dimension, e.g., $\mathbb{R}^{n\geq 4}$. Intrinsically, the (fractal) space-time is perceived "almost" as the usual fourdimensional space.

Besides such considerations, J. A. Wheeler [12], among others, has emphasised the role of *observer-participancy.* In the context of what is considered by the Einstein-Podolsky-Rosen argument [13] as "incompleteness" of quantum theory, A. Peres and W. H. Zurek [14, 15] and J. Rothstein [16] have attempted to relate quantum complementarity to Gödel-type incompleteness.

In what follows, the intrinsic-extrinsic concept will be made precise in an *algorithmic* context, thereby closely following E. F. Moore [6]. The main reason for the algorithmic approach is that algorithmic universes (or, equivalently, formal systems) are the royal road to the study of undecidability. The intrinsic-extrinsic concept will be applied to investigate *computational complementarity* and *intrinsic indeterminism* both in the algorithmic context.

2 Gedankenexperiments on finite automata

In a groundbreaking study [6], Edward Moore analysed two kinds of *Gedankenexperiments* on finite automata, which will be slightly adapted for the present purposes. In both cases, the automaton is treated as a "black box" in the following sense:

(i) only the input and output terminals of the automaton are accessible. The experimenter is allowed to perform experiments *via* these interfaces in the form of stimulating the automaton with input sequences and receiving output sequences from the automaton. The experimenter is not permitted to "open up" the automaton, but

(ii) the transition and output table (diagram) of the automaton (in its reduced form) is known to the experimenter (or, if you prefer, is given to the experimenter by some "oracle").

The most important problem, among others, is the *distinguishing problem:* it is known that an automaton is in one of a particular class of internal states: find that state.

In the first kind of experimental situation, only a *single* copy of the automaton is accessible to the experimenter. The second type of experiment operates with an *arbitrary number* of automaton copies. Both cases will be discussed in detail below.

If the input is some *predetermined* sequence, one may call the experiment a *preset experiment*. If, on the other hand, (part of) the input sequence depends on (part of) the output sequence, i.e., if the input is *adapted* to the reaction of the automaton, one may call the experiment an *adaptive experiment.* We shall be mostly concerned with preset experiments, yet adaptive experiments can be used to solve certain problems with automaton propositional calculi.

Research along these lines has been pursued by S. Ginsburg [17], A. Gill [18], J. H. Conway [30] and W. Brauer [31].

2.1 Single-automaton configuration

In the first kind of Gedankenexperiment, only *one single* automaton copy is presented to the experimenter. The problem is to determine the initial state of the automaton, provided its transition and output functions are known (distinguishing problem). In a typical experiment, the automaton is "feeded" with a sequence of input symbols and responds by a sequence of output symbols. An input-output analysis then reveals information about the automaton's original state.

Assume for the moment that such an experiment induces a state transition of the automaton. I.e., after the experiment, the automaton is not in the original initial state. In this process a loss of potential information about the automaton's initial state may occur. In other words: certain measurements, while measuring some particular feature of the automaton, may make impossible the measurement of other features of the automaton. This irreversible change of the automaton state is one aspect of the "observer-participancy" in the single-automaton configuration. (This is not the case for the multi-automaton situation discussed below, since the availability of an arbitrary number of automata ensures the possibility of an arbitrary number of measuring processes.)

In developing the intrinsic concept further, the automaton and the experimenter are "placed" into a *single* "meta"-automaton. If the experimenter reacts mechanically, this can be readily achieved by simulating both the original finite deterministic "black box" automaton as well as the experimenter and their interplay by a universal automaton. One can imagine such a situation as one subprogram checking another subprogram, also including itself. For an illustration, see Fig. 1.

In certain cases it is necessary to iterate this picture in the following way. Suppose, for instance, the experimenter attempts a *complete* intrinsic description. Then, the experimenter has to give a complete description of his own intrinsic situation. In order to be able to model the own intrinsic viewpoint, the experimenter has to introduce an or system which is a *replica* of its own universe. This amounts to substituting the "meta"-automaton for the automaton in Fig. 1. Compare also a drawing by O. E. Rössler [3], Fig. 2, where "$\approx$" stands for the interface, which is denoted by the symbols "$\leftrightarrow$" throughout this article. Yet, in order to be able to model intrinsic viewpoint of a new experimenter in this new universe, this new experimenter has to introduce another system which is a *replica* of its own universe, $\ldots$, resulting in an iteration *ad infinitum.* One may conjecture that an observer in a hypothetical universe corresponding to the "fixed point" or "invariant set" of this process has complete self-comprehension; see Fig. 3. Of course, in general this observer cannot be a finite observer: a complete description would only emerge in the limit of infinite iterations (cf. K. Popper's "paradox of Tristram Shandy"). Finite observers cannot obtain complete self-comprehension.

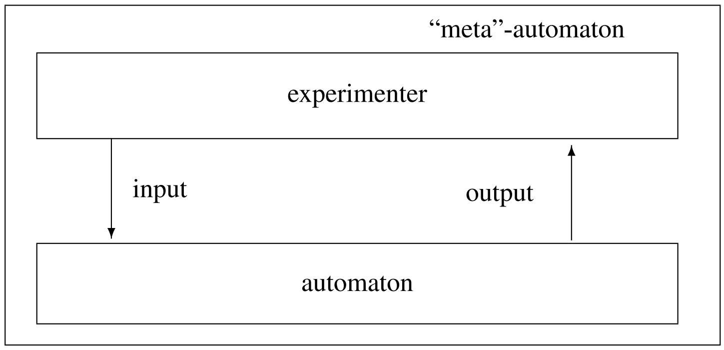

Figure 1: Schematic diagram of an experiment on a single automaton, both taking place in a "meta"-automaton.

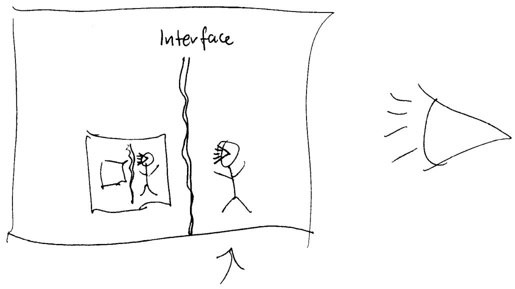

Figure 2: Author's notes from a seminar talk by O. E. Rössler.

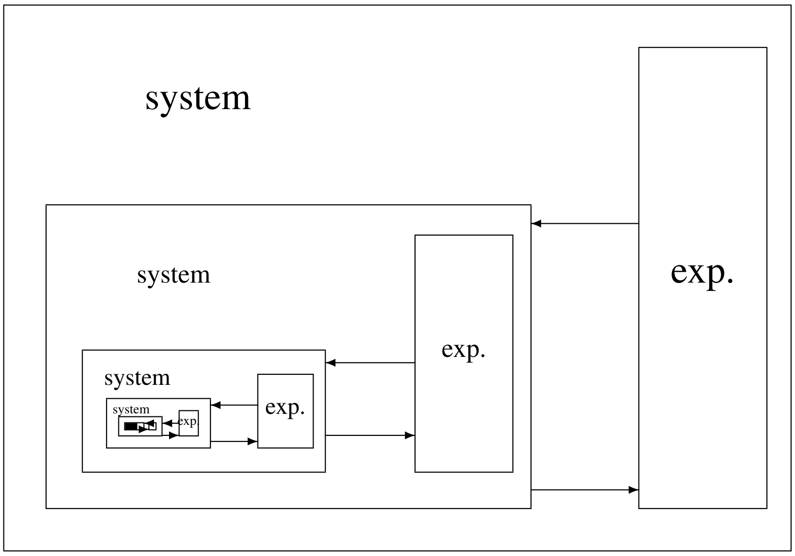

Figure 3: Hierarchy of intrinsic perception.

2.2 Multi-automata configuration

The second kind of experiment operates with an *arbitrary number* of automaton copies. One automaton is a copy of another if both automata are isomorphic and if both are in the same initial state. With this configuration the experimenter is in the happy condition to apply as many input sequences to the original automaton as necessary. In a sense, the observer is not bound to "observer-participancy," because it is always possible to "discard the used automaton copies," and take a "fresh" automaton copy for further experiments. For an illustration, see Fig. 4.

3 Definition

In the foregoing section, important features of the extrinsic-intrinsic concept have been isolated in the context of finite automata. A generalisation to arbitrary physical systems is straightforward. The features will be summarised by the following definition. (Anything on which experiments can be performed will be called *system*. In particular, finite automata are systems.)

An *intrinsic* quantity is associated with an experiment
(i) performed on a *single copy* of the system,
(ii) with the experimenter being part of the system.

An *extrinsic* quantity, denoted by a tilde sign "$\widetilde{\ }$" is associated with an experiment
(i) utilising, if necessary, an *arbitrary number of copies* of the system,

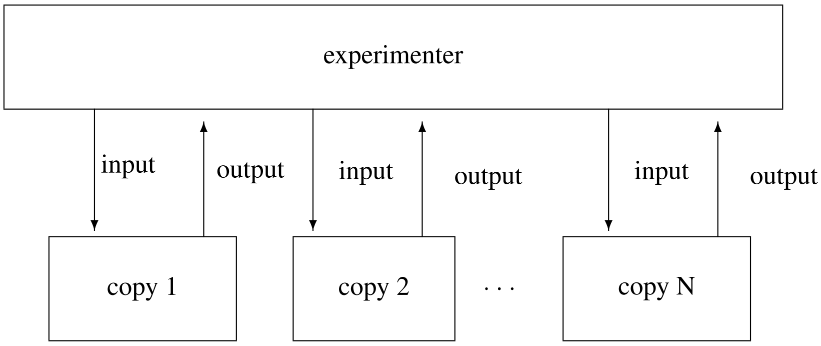

Figure 4: Schematic diagram of an experiment on an arbitrary number of identical automaton copies.

(ii) with the experimenter not being part of the system.

One may ask whether, intuitively, the extrinsic point of view might be more appropriately represented by, stated pointedly, the application of a "can-opener" for the "black box" to see "what's really in it." Yet, while the physical realisation might be of some engineering importance, the primary concern is the phenomenology (i.e., the *experimental performance* of the system) and not how it is constructed. In this sense, the technological base of the automaton is irrelevant. For the same reason, i.e., because this is irrelevant to phenomenology, it is not important whether the automaton is in its minimal form.

The requirement that in the extrinsic case an *arbitrary* number of system copies is available is equivalent to the statement that *no interaction takes place between the experimenter and the system.* (The reverse information flow from the observed system to the experimenter is necessary.) This results in a one-way information flow in the extrinsic case:

$$\text{system} \ \underset{\not\Longleftarrow}{\Longrightarrow} \ \text{experimenter} \qquad ,$$

and a two-way information flow in the intrinsic case:

$$\text{system} \Longleftrightarrow \text{experimenter} \qquad .$$

An information "backflow" makes possible the application of diagonalization techniques and also results in complementarity, which might be seen as a "poor man's version of diagonalization."

The definition applies to physical systems as well as to logics and (finite) automata. Automaton worlds provide an ideal "playground" for the study of certain algorithmic features related to undecidability, such as "computational complementarity" and "intrinsic indeterminism." The *extrinsic-intrinsic problem* is the interrelation between extrinsic and intrinsic entities.

4 Complementarity

The input-output analysis of finite automata yields a fresh insight into the quantum mechanical feature of complementarity on a very elementary level. Conversely, the Copenhagen interpretation of quantum mechanics [19, 20] can be applied for the analysis of automata. To substantiate this claim it is necessary to interrelate two strains of investigation: (i) the lattice theoretic [21] approach for a representation of quantum physics, pioneered by G. Birkhoff and J. von Neumann [22] and later extended to the calculus of propositions [23, 24] and orthomodular logic [25, 26, 27, 28]; (ii) the theory of finite automata, in particular of Moore and Mealy automata [6, 29, 30, 31]. Computational complementarity in the automata context has been first investigated by E. F. Moore in his article *Gedanken-Experiments on Sequential Machines* [6]. Informally stated, measurement of one aspect of an automaton makes impossible measurement of another aspect and *vice versa*. The name *computational complementarity* is due to D. Finkelstein [32, 33], who also made the first attempt to construct logics from experimentally obtained propositions about automata; see also the more recent investigation by A. A. Grib and R. R. Zapatrin [34]. The following investigation has been carried out independently. Although the goals are very similar, the methods and techniques used here differ from the ones used by previous authors.

The investigation is based on the construction of primitive *experimental statements* or *propositions*. Then the *structure* of these propositions will be discussed, thereby defining algebraic relations and operations between the propositions. Although specific classes of finite automata will be analysed, these considerations apply to universal computers as well. (Finite automata can be simulated on universal computers.)

4.1 Finite automata

A *finite (i,k,n)-automaton* has a finite number of i internal states, k input and n output symbols. It is characterised by its transition and output functions δ and o, which are often represented by transition and output tables and by a diagram. For an example see below. The output function of a *Moore-type automata* depends solely on its internal state, whereas the output function of *Mealy-type automata* depends on the input and the internal state.

4.2 Automaton propositional calculi

A finite automaton will be treated as a "black box," whose transition and output tables (i.e., informally speaking, its "intrinsic machinery") are given in advance but *whose initial state is unknown.* Only *a single* copy of the automaton will be made available to the experimenter. The automaton is "feeded" with certain input sequences from the experimenter and responds with certain output sequences. We shall be interested in the *distinguishing problem*: *"identify an unknown initial state."*

Consider propositions of the form

"`the automaton is in state` a_j"

with $(1 \leq j \leq i)$. Propositions can be composed to form expressions of the form "`the automaton is in state` a_j `or in state` a_m `or in state` $a_l \cdots$."

Any proposition composed by propositions can be represented by a set. E.g., the above statement "`the automaton is in state` a_j `or in state` a_m `or in state` $a_l \cdots$" represents the set $\{j,m,l,\ldots\}$. The element **1** is given by the set of *all* states $\{1,2,\ldots,i\}$. This corresponds to a proposition which is always satisfied:

"`the automaton is in some internal state`"

The element **0** is given by the *empty* set $\emptyset$ (or $\{\}$). This corresponds to a proposition which is false (by definition the automaton has to be in *some* internal state):

"`the automaton is in no internal state`"

The class of all propositions and their relations will be called *automaton propositional calculus* and denoted by $\mathfrak{A}$. Each particular outcome which, if defined, has the value `TRUE` or `FALSE`, shall be called "event." In this sense, an automaton propositional calculus, just as the quantum propositional calculus, is obtained *experimentally*. It consists of all potentially measurable *elements of the automaton reality* and their logical structure, with the implication as order relation.

The elementary propositions can be conveniently constructed by a partitioning of automaton states generated from the input-output analysis of the automaton as follows: Let $w = s_1 s_2 \cdots s_k$ be a sequence of input symbols,

$$a_{i,w} = a_i \delta_{s_1}(a_i)\delta_{s_2}(\delta_{s_1}(a_i)) \cdots \delta_{s_k}(\cdots \delta_{s_1}(a_i)\cdots) \tag{1}$$

and

$$z = o(a_{i,w}) = o(a_i)o(\delta_{s_1}(a_i))o(\delta_{s_2}(\delta_{s_1}(a_i))) \cdots o(\delta_{s_k}(\cdots \delta_{s_1}(a_i)\cdots)) \quad . \tag{2}$$

Let

$$\alpha_z^w = \{a_i \mid o(a_{i,w}) = z\} \tag{3}$$

be the set of initial states which, on some fixed input sequence w yield some fixed output sequence $z = t_0 t_1 t_2 \cdots t_k$. I.e., α_z^w is the equivalence class of propositions identifyable by input w and output z. The elements $\{\alpha_z^w\}$ of the partition

$$v(w) = \bigcup_z \{\alpha_z^w\} \tag{4}$$

define the equivalence classes of propositions identifiable by input w and output z.

$$V = \bigcup_w v(w) = \{v(\emptyset), v(s_1), \ldots, v(s_k), v(s_1 s_2), \ldots\} \tag{5}$$

is the set of partitions.

Let p_i be propositions of the form "`the automaton is in state` a_i." The proposition

$$p_1 \vee p_2 \tag{6}$$

(interpretable as "p_1 or p_2") defines a proposition of the form "`the automaton is in state` a_1 `or in state` a_2" (or the set theoretic union "$p_1 \cup p_2$") if and only if there exist input sequences $s_j \cdots s_m$ such that $p_1 \vee p_2$ is identified by the partition $v(s_j \cdots s_m)$.

The proposition

$$p_j \wedge p_m \tag{7}$$

(interpretable as "p_j and p_m") defines a proposition of the form "p_j `and` p_m" (or the set theoretic intersection "$p_j \cap p_m$") if and only if there exist input sequences $s_j \cdots s_m$ such that $p_1 \wedge p_2$ is identified by the partition $v(s_j \cdots s_m)$.

The complement

$$\neg p_1 \tag{8}$$

(or p_1') of a proposition p_1 (has the meaning of "not p_1" and) is defined if and only if

$$\begin{aligned} p_1 \wedge \neg p_1 &= \mathbf{0} \\ p_1 \vee \neg p_1 &= \mathbf{1} \end{aligned}$$

(or, with the propositions p_1 and $\neg p_1 = p_j$ expressed as sets, $p_1 \cap p_j = \mathbf{0} = \emptyset$ and $p_1 \cup p_j = \mathbf{1} = \{1, 2, \ldots, i\}$), and there exist input sequences $s_j \cdots s_m$ such that $\neg p_1$ is a proposition identified by the partition $v(s_j \cdots s_m)$.

A *partial order relation* $p_j \preceq p_m$, or

$$p_j \rightarrow p_m \tag{9}$$

(with the interpretation "p_j implies p_m," or with "whenever p_j is true it follows that p_m is true, too") is defined if and only if p_j `implies` p_m, and there exist input sequences $s_j \cdots s_m$ such that p_j and p_m are propositions identified by the partition $v(s_j \cdots s_m)$. The partial order relation can be conveniently represented by drawing the Hasse diagram thereof. This can be done by proceeding in two steps. First, the Boolean lattices of propositional structures based on all relevant state partitions $v(w)$ are constructed. Then, the union of all these Boolean subalgebras renders the complete partial order of the automaton propositional calculus. This can also be understood graph theoretically [38, 39]. A *Mathematica* package by Ch. Strnadl [40] can be obtained from the author.

4.3 Example

For an explicit model of a non distributive and modular automaton propositional calculus consider the transition and output tables 1 of a (3,3,2)-automaton. Its diagram is drawn in Fig. 5.

Input of 1, 2 or 3 steers the automaton into the respective state. At the same time, the output of the automaton is 1 only if the guess is a "hit," i.e., if the automaton was in that state. Otherwise the output is 0. After the measurement, the automaton is in a definite state, i.e., the state corresponding to the input symbol. If the guess has not been a "hit," the information about the initial automaton state is lost. Therefore, the experimenter has to decide before carrying out the measurement which one of the following hypotheses should be tested (in short-hand notation, "$\{1\}$" stands for "`the automaton is in state 1`" *et cetera*): $\{1\} = \neg\{2,3\}, \{2\} = \neg\{1,3\}, \{3\} =$

	1	2	3
δ_1	1	1	1
δ_2	2	2	2
δ_3	3	3	3
o_1	1	0	0
o_2	0	1	0
o_3	0	0	1

Table 1: Transition and output table of a (3,2,2)-automaton of the Mealy type.

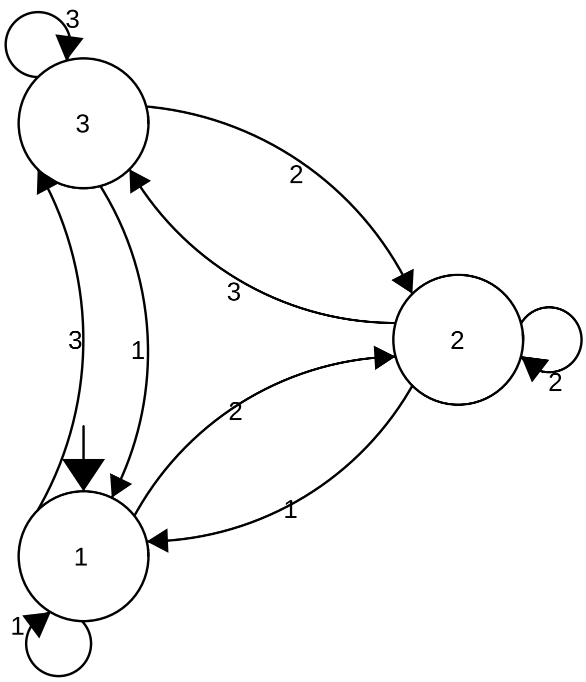

Figure 5: Diagram of a (3,2,2)-automaton of the Mealy type featuring computational complementarity.

$\neg\{1,2\}$. Measurement of either one of these three hypotheses (or their complement) makes impossible measurement of the other two hypotheses.

No input, i.e., the empty input string $\emptyset$, identifies all three internal automaton states. This corresponds to the trivial information that the automaton is in *some* internal state. Input of the symbol 1 (and all sequences of symbols starting with 1) distinguishes between the hypothesis $\{1\}$ (output "1") and the hypothesis $\{2,3\}$ (output "0"). Input of the symbol 2 (and all sequences of symbols starting with 1) distinguishes between the hypothesis $\{2\}$ (output "1") and the hypothesis $\{1,3\}$ (output "0"). Input of the symbol 3 (and all sequences of symbols starting with 1) distinguishes between the hypothesis $\{3\}$ (output "1") and the hypothesis $\{1,2\}$ (output "0"). The propositional calculus is thus defined by the partitions

$$
\begin{aligned}
v(\emptyset) &= \{\{1,2,3\}\} \quad , &(10)\\
v(1) &= \{\{1\},\{2,3\}\} \quad , &(11)\\
v(2) &= \{\{2\},\{1,3\}\} \quad , &(12)\\
v(3) &= \{\{3\},\{1,2\}\} \quad . &(13)
\end{aligned}
$$

It can be represented by the lattice structure of Fig. 6. This lattice is of the "Chinese latern" $MO3$ form. It is non distributive, and it is a pasting of three Boolean algebras 2^2.

The obtained intrinsic propositional calculus in many ways resembles the lattice obtained from photon polarisation experiments or from other incompatible quantum measurements. Consider an experiment measuring photon polarisation. Then, three propositions of the form "`the photon has polarisation` p_{ϕ_1}," ($i = 1,2,3$), cannot be measured simultaneously for the angles $\phi_1 \neq \phi_2 \neq \phi_3 (\text{mod} \pi)$. An irreversible measurement of one direction of polarisation would result in a state preparation, making impossible measurement of the other directions of polarisation, and resulting in a propositional calculus of the "Chinese latern" form $MO3$.

The propositional calculi $\mathfrak{F}_i$ of all Mealy-type automata with i internal states can be constructed by combinatorical arguments [41]. Fig. 7 shows $\mathfrak{F}_4$, the Hasse diagrams of generic intrinsic propositional calculi of Mealy automata up to 4 states.

4.4 The inverse problem

The previous paragraphs concentrated on the construction of a suitable propositional calculus from the input-output analysis of an automaton. The inverse problem is the construction of suitable automata which correspond to (orthomodular) lattices, in particular to subalgebras of Hilbert lattices. Stated differently: *"given an arbitrary orthomodular (subalgebra of a Hilbert) lattice $\mathfrak{L}$; is it possible to construct an automaton propositional calculus $\mathfrak{A}$ realising $\mathfrak{L}$?"* If, as will be shown below, (for finite lattices) the question can be decided positively and constructively, then one obtains an explicit automaton model for every arbitrary quantum system (but not *vice versa*).

Let an *orthomodular lattice* be a lattice satisfying the orthomodular law, and let a *Hilbert lattice* be the lattice of all closed subspaces of a Hilbert space, with the "infimum" operator defined by the intersection of subspaces, the "supremum" operator

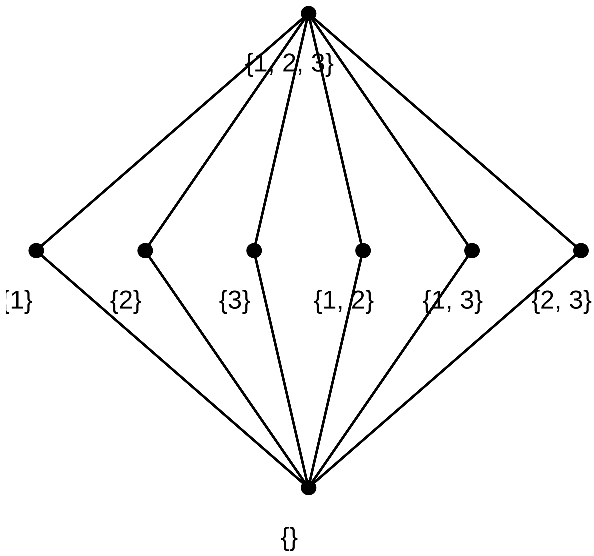

Figure 6: Lattice $MO3$ of intrinsic propositional calculus of a (3,2,2)-automaton of the Mealy type.

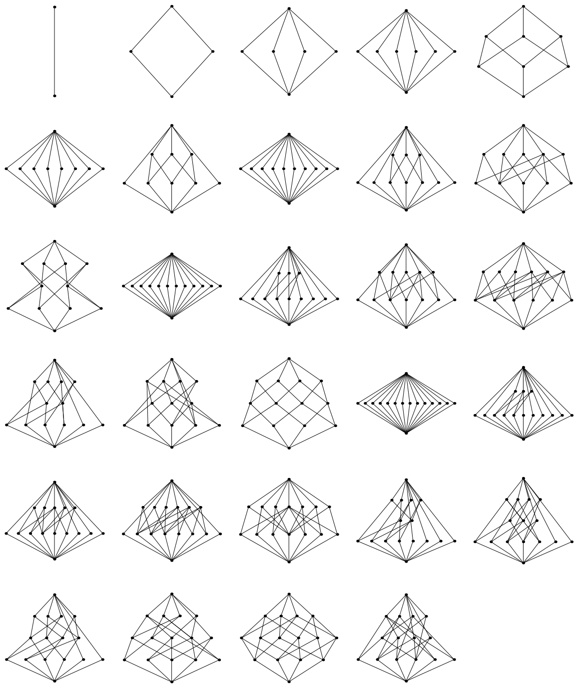

Figure 7: The class $\mathfrak{F}_4$ of non isomorphic Hasse diagrams of the intrinsic propositional calculi of generic 4-state automata of the Mealy type.

defined by the closure of the linear span of subspaces and the orthocomplement defined by the orthogonal subspace. Any finite ("finite" means that the lattice has a finite number of elements) orthomodular lattice is isomorphic (1-1 translatable) to some finite (lattice) automaton propositional calculus. I.e.,

$$\text{finite orthomodular lattice} \begin{array}{c}\Rightarrow\\ \Leftarrow\end{array} \text{finite automaton propositional calculus} \qquad (14)$$

Threfore, any finite orthomodular subalgebra of a Hilbert lattice is isomorphic (1-1 translatable) to some finite automaton propositional calculus. I.e.,

$$\left\{ \begin{array}{c} \text{finite orthomodular subalgebra} \\ \text{of Hilbert lattice (quantum logic)} \end{array} \right\} \begin{array}{c}\Rightarrow\\ \Leftarrow\end{array} \text{finite automaton propositional calculus} \quad (15)$$

An actual proof of these statements is too technical and will be given elsewhere [41]. It makes use of the fact that every orthomodular lattice is a pasting of its maximal Boolean subalgebras, also called *blocks* [25, 42]. These blocks can be elegantly represented by sets of partitions of automata states, because "at face value," every automaton state partition $v(\cdots)$ with n elements generates a Boolean algebra 2^n. If one identifies these Boolean algebras with blocks, the set of automaton state partitions V represents a complete family of blocks of the automaton propositional calculus.

4.5 Discussion

Strictly speaking, automaton models for quantum systems correspond to nonlocal hidden variable models. The "hidden" physical entities are the "true" initial states of automata.

It is not entirely unreasonable to speculate about logico-algebraic structures of automaton universes in general. To put it pointedly, one could ask *"how would creatures embedded in a universal computer perceive their universe?"* The lattice-theoretic answer might be as follows. Let $\mathfrak{F}_i$ stand for the family of all intrinsic propositional calculi of automata with i states. From the point of view of logic, the intrinsic propositional calculi of a universe generated by universal computation is the limiting class $\lim_{n\to\infty} \mathfrak{F}_n$ of all automata with $n \to \infty$ states. Since $\mathfrak{F}_1 \subset \mathfrak{F}_2 \subset \mathfrak{F}_3 \subset \cdots \subset \mathfrak{F}_i \subset \mathfrak{F}_{i+1} \subset \cdots$, this class "starts with" the propositional calculi represented by Fig. 7, p. 14.

It is tempting to speculate that we live in a computer generated universe. But then, if the "underlying" computing agent were universal, *there is no a priori reason to exclude propositional calculi even if they do not correspond to an orthomodular subalgebra of a Hilbert lattice.* I.e., to test the speculation that we live in a universe created by universal computation, we would have to look for phenomena which correspond to automaton propositional calculi not contained in the subalgebras of some Hilbert space — such as, for instance, the one represented by Fig. 8, p. 16, which was obtained from the state partition $\{\{\{1\},\{2\},\{3,4\}\},\{\{1\},\{2,4\},\{3\}\},\{\{1,2\},\{3\},\{4\}\},\{\{1,3\},\{2\},\{4\}\}\}$.

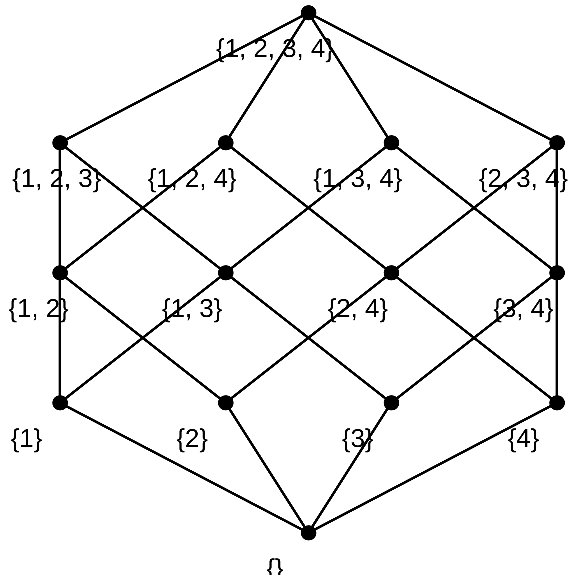

Figure 8: Hasse diagram of an algebraic structur which is neither a lattice nor a partial order.

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